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William Nicholas Roseveare, M.A., Professor of Mathematics, Natal University College, Pietermaritzburg, Natal;
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William Samuel Warren, 11 East Grove, Cardiff; and
Arthur Harry Witney, 5 Albert Road, Brockley, London, S.E. 4,

were balloted for and duly elected Fellows of the Society.

The following candidate was proposed for election as Fellow of the Society, the name of the proposer from personal knowledge being appended:—

Louis Melville Milne-Thomson, B.A., Lecturer in Mathematics, Royal Naval College, Greenwich, S.E. 10 (proposed by R. L. Waterfield).

Sixty-one presents were announced as having been received since the last meeting, including, amongst others:—

Sproul Observatory Publications, No. 6 ("The Parallaxes of 50 Stars.")

The Progression of Stellar Velocity with Absolute Magnitude.
By A. S. Eddington, F.R.S., and Allie Vibert Douglas.

1. Several investigators have shown that there is a correlation between the velocities and luminosities of stars, intrinsically faint stars moving faster on the average than bright stars. It is of great importance to determine the amount of the change of velocity per magnitude. Spectroscopic parallaxes, now published in considerable numbers, provide good material for this determination. When used in conjunction with radial velocities no special difficulty arises; but when used in conjunction with proper motions some care is necessary, as the crude direct method is likely to give fallacious results.

The spectroscopic parallaxes, besides affording the absolute magnitudes, have to be used in order to convert the angular proper motions into km. per sec. When by accidental error too bright an absolute magnitude is assigned, the star is located too far away and the deduced linear speed is too great; similarly, if the assigned magnitude is too faint, the linear speed is too small. There is thus a tendency falsely to attribute high velocities to (supposed) bright stars and low velocities to

faint stars. To take an extreme case, if all the stars observed were of the same true brightness, the uncorrected investigation would show a decrease of velocity in the ratio $(2{\cdot}512)^{-\frac{1}{5}}$, or 0·63 per magnitude. In practical cases the effect of the accidental errors of the magnitudes is smaller; but the *accidental* error of the magnitudes produces a *systematic* error in the deduced change of velocity which is usually considerable, and it may even mask the true progression of velocity with brightness, which is in the opposite direction.

The purpose of this paper is to show how to eliminate the effect of accidental error in the spectroscopic absolute magnitudes. If the degree of precision of the observations is known, the true progression of velocity with magnitude can be obtained by the method here given. We have applied the theory to a discussion of the largest homogeneous group of stars available, viz. the giants of type K.

2. Let m be the true absolute magnitude,
μ the observed absolute magnitude,
v the true linear velocity,
v the calculated linear velocity (*i.e.* using the parallax corresponding to μ instead of m).

Then

$$v = ve^{-q(\mu - m)} \quad . \quad . \quad . \quad . \quad . \quad (1)$$

where $q = 0{\cdot}4605$. The value of q is found by considering that if the error $\mu - m$ were 5 magnitudes, the distance and linear speed would be altered in the ratio 10. Hence $e^{5q} = 10$.

In a homogeneous group of stars such as the K giants, the absolute magnitudes are usually found to be distributed according to a frequency-law agreeing closely with the error-law. Accordingly we shall have

$$\text{Frequency of } m \text{ to } m + dm = \frac{k}{\sqrt{\pi}} e^{-k^2 m^2}\, dm \quad . \quad . \quad (2)$$

$$\text{Frequency of } \mu \text{ to } \mu + d\mu = \frac{k'}{\sqrt{\pi}} e^{-k'^2 \mu^2}\, d\mu \quad . \quad . \quad (3)$$

$$\text{Frequency of an error } (\mu - m) \text{ to } (\mu + d\mu - m) = \frac{h}{\sqrt{\pi}} e^{-h^2(\mu - m)^2}\, d\mu \quad (4)$$

and by the well-known law of addition of accidental errors

$$\frac{1}{k'^2} = \frac{1}{k^2} + \frac{1}{h^2} \quad . \quad . \quad . \quad . \quad . \quad (5)$$

The mean absolute magnitude of the group is here taken as the zero-point from which m and μ are reckoned. Here h is the measure of precision of the observations of absolute magnitude, and k' must be calculated from the statistics under discussion.

Suppose that within the range of magnitude covered by the group, the variation of velocity with absolute magnitude can be represented by

$$v = a + bm + cm^2 + \ldots \quad . \quad . \quad . \quad . \quad . \quad (6)$$

Our problem is to determine the coefficients of this formula from statistics of μ and v. By (1), (2), and (4) the sum of the v's for all stars between μ and $\mu+d\mu$ will be given by

$$\Sigma v = \int_{-\infty}^{\infty} \frac{h}{\sqrt{\pi}} e^{-h^2(\mu-m)^2}\, d\mu \cdot \frac{k}{\sqrt{\pi}} e^{-k^2m^2}\, dm \,.\, (a+bm+cm^2) e^{-q(\mu-m)} \qquad (7)$$

The integration is for all values of m (erroneously observed as μ). This reduces to

$$\Sigma v = d\mu \cdot \frac{hk}{\pi} \exp\left\{\frac{\frac{1}{4}q^2 - k^2(h^2\mu^2+q\mu)}{h^2+k^2}\right\} \int_{-\infty}^{\infty} (a+bm+cm^2) \exp \left\{-(h^2+k^2)\left(m - \frac{h^2\mu+\frac{1}{2}q}{h^2+k^2}\right)^2\right\} dm$$

$$= d\mu \cdot \frac{hk}{\pi} \exp\left\{\frac{\frac{1}{4}q^2 - k^2(h^2\mu^2+q\mu)}{h^2+k^2}\right\} \left\{a + b\frac{(h^2\mu+\frac{1}{2}q)}{h^2+k^2} + c\left(\frac{h^2\mu+\frac{1}{2}q}{h^2+k^2}\right)^2 + \frac{c}{2(h^2+k^2)}\right\} \frac{\sqrt{\pi}}{\sqrt{(h^2+k^2)}}.$$

The number of stars contributing to this sum is by (3) and (5).

$$\frac{d\mu}{\sqrt{\pi}} \frac{hk}{\sqrt{(h^2+k^2)}} \exp\left(-\frac{h^2k^2\mu^2}{h^2+k^2}\right).$$

Hence by division the mean value of v is

$$v = \exp\left\{\frac{\frac{1}{4}q^2 - k^2q\mu}{h^2+k^2}\right\}\left(a + b\frac{h^2\mu+\frac{1}{2}q}{h^2+k^2} + \ldots\right) \quad . \quad . \quad (8)$$

At present the data are not likely to be sufficiently extensive to justify an attempt to determine c, and we shall therefore confine attention to the coefficients a and b.

The following practical procedure is indicated:—Having calculated from the observational data the mean value v for a suitable number of mean observed magnitudes μ, we next calculate

$$v' = v \exp\left\{-(\tfrac{1}{4}q^2 - k^2q\mu)/(h^2+k^2)\right\} \quad . \quad . \quad . \quad (9)$$

and find by least squares the best representation of v' as a linear function of μ, viz.

$$v' = a' + b'\mu.$$

Then by (8)

$$a' = a + \tfrac{1}{2}qb/(h^2+k^2) \qquad b' = h^2b/(h^2+k^2) \quad . \quad . \quad (10)$$

so that a and b can be found.

An example of this method is given in § 4.

3. An alternative method is to assume for the law expressing increase of velocity with magnitude the form

$$\log_e v = a + bm.$$

The factor $(a+bm+cm^2)$ in (7) must now be replaced by e^{a+bm}. Carrying out the reductions by the same method as before we find that

$$\log_e v = a' + b'\mu$$

where

$$a' = a + (b+q)^2/4(h^2+k^2) \qquad b' = (bh^2 - qk^2)/(h^2+k^2).$$

It may be noticed that the progression of velocity with magnitude, b', shown by the crude statistics will not even be in the same direction as the true progression, b, unless $b/q > k^2/h^2$. It would be easy to find cases in which this inequality is not satisfied.

The importance of the correction arises from the fact that we have generally to deal with a group of stars extending over a very limited range of magnitude in which the actual dispersion of the magnitudes is not incomparably greater than the dispersion due to observational error. It is now generally understood that the giant and dwarf divisions must be treated separately, as it is not anticipated that a continuous formula would be likely to cover both groups together.

4. The following investigation relates to the stars from type G8 to K2 inclusive, brighter than absolute magnitude $3^m{\cdot}5$ (which seems to be the dividing line between giants and dwarfs) contained in *Astrophysical Journal*, 53, p. 13 (1921) "Parallaxes of 1646 Stars," by Adams, Joy, Strömberg, and Burwell. Proper motions were taken from Boss's Preliminary General Catalogue; stars not included in Boss's Catalogue were omitted, as it was thought that they might have been selected for special reasons. 287 stars remained.

In order to eliminate the solar motion, the cross-component of proper motion at right angles to the direction towards the solar apex was calculated for each star. These were converted into linear velocity by using the spectroscopic parallaxes. The stars were arranged in order of absolute magnitude and divided into five groups containing approximately equal numbers. The mean linear cross-velocity v and the mean absolute magnitude μ for each group are given in Table I., the unit of v being one astronomical unit per annum, or 4·74 km. per sec. In order to reduce the accidental effect of a few excessive motions the table also contains the mean values when (1) the three largest velocities in each group, and (2) the six largest velocities in each group are omitted.

TABLE I.

Limits of Group.	Mean μ.	v (all).	v (−3).	v (−6).
m m	m			
−3·6 − +0·4	−0·27	3·35	2·67	2·19
+0·5 − +0·8	+0·67	3·97	3·36	3·06
+0·9 − +1·2	+1·05	3·43	2·86	2·40
+1·3 − +1·7	+1·49	3·57	3·13	2·80
+1·8 − +3·4	+2·37	3·68	3·08	2·68

The mean magnitude for the whole group is $+1^m{\cdot}01$, and the average deviation is $\pm 0^m{\cdot}691$. Hence $k' = 1/{\cdot}691 \times \sqrt{\pi} = 0{\cdot}817$.

We have assumed that the probable error of a magnitude determination is $\pm 0^{m}\cdot 3$.* Hence $h = \cdot 477/0\cdot 3 = 1\cdot 59$.

By (5) we then find $k = 0\cdot 952$.

The values of $v'/v = \exp\{-(\frac{1}{4}q^2 - k^2 q(\mu - 1\cdot 01))/(h^2 + k^2)\}$ for the five groups are respectively

$$\cdot 844,\ \cdot 946,\ \cdot 991,\ 1\cdot 045,\ 1\cdot 164.$$

Hence we obtain the following table:—

TABLE II.

Mean μ.	v' (all).	v' (−3).	v' (−6).
m			
−0·27	2·83	2·25	1·85
+0·67	3·76	3·18	2·89
+1·05	3·40	2·83	2·38
+1·49	3·73	3·27	2·92
+2·37	4·28	3·59	3·11

The linear formula best representing these values is $v' = a' + b'(\mu - 1\cdot 01)$, with

$$a' = 3\cdot 56,\quad 2\cdot 98,\quad 2\cdot 60$$
$$b' = \cdot 501,\quad \cdot 479,\quad \cdot 446.$$

Hence by (10), $v = a + b(m - 1\cdot 01)$, with

$$a = 3\cdot 52,\quad 2\cdot 94,\quad 2\cdot 56$$
$$b = \cdot 681,\quad \cdot 652,\quad \cdot 606.$$

The values in the last two columns, corresponding to the omission of the largest velocities, are naturally smaller than those for which all velocities are included, and they should be multiplied by the proper compensating factor. If out of N velocities the n highest are omitted, the error-distribution is cut short at a velocity x ($= kv$) given by

$$\int_x^\infty e^{-x^2}dx = \frac{n}{N}\int_0^\infty e^{-x^2}dx \quad . \quad . \quad . \quad . \quad (11)$$

And the mean velocity is then

$$\int_0^x xe^{-x^2}dx \div \int_0^x e^{-x^2}dx$$

instead of

$$\int_0^\infty xe^{-x^2}dx \div \int_0^\infty e^{-x^2}dx.$$

The compensating factor is thus found to be

$$\frac{1}{1 - e^{-x^2}} \cdot \frac{N - n}{N}$$

* The authors give the probable error as $\pm 0^{m}\cdot 4$; the lower value adopted here will at any rate not exaggerate the effect which we are studying.

where x is found by solving (11). This factor is 1·12 when the three highest are omitted, and 1·22 when the six highest are omitted. The corrected values are thus

$$a = 3\cdot52, \quad 3\cdot29, \quad 3\cdot12$$
$$b = \cdot681, \quad \cdot730, \quad \cdot739.$$

Multiplying by 4·74 to reduce to km. per sec., and measuring the absolute magnitude from zero instead of $1^m\cdot01$ the final formulæ are

$v = 13\cdot4 + 3\cdot22m$ km. per sec. (all stars)
$v = 12\cdot1 + 3\cdot46m$ „ „ (15 rejected)
$v = 11\cdot3 + 3\cdot50m$ „ „ (30 rejected).

The three formulæ agree so nearly that it is scarcely necessary to decide which is the most trustworthy; but as a general rule we might expect that the omission of the three highest velocities out of fifty would remove all difficulty from excessive velocities of exceptional stars, and we accordingly take the middle solution as definitive. Of course, the formula must not be extrapolated beyond the range for which it is derived, viz. from $-0^m\cdot3$ to $+2^m\cdot4$. In this range the formula gives an increase of velocity from 11·1 km. per sec. to 20·4 km. per sec., a ratio of 1·84.

It is of interest to compare this result with that of Adams, Strömberg, and Joy, who have worked on the same material.* Their division most nearly corresponding to ours is the group of giants from Ko to K9. Their formulæ gave a velocity ratio for a change of 2·7 magnitudes

from proper motions 1·19
from radial velocities 1·93.

Our result, 1·84, agrees with the radial velocities, and at the same time we see why the uncorrected result from the proper motions was so small. An examination of our Table I. reveals no clear indication of a change of v with μ; it is only when the masking influence is removed that we are able to obtain the progression of v with m.

5. Calculating the probable error of our determination of b in the usual way, we find that the increase of velocity is

$$3\cdot46 \pm 0\cdot65 \text{ km. per sec. per magnitude.}$$

The probable error is as low as could be anticipated theoretically,† and, so far as internal agreement is concerned, the result would appear to rank as a good determination. But a little consideration shows that its trustworthiness depends on the accuracy of the assumed probable error of the spectroscopic magnitudes. We have made another solution assuming a probable error $\pm 0^m\cdot4$ instead of $\pm 0^m\cdot3$. The result (rejecting three highest velocities in each group) is now

$$v = 7\cdot7 + 7\cdot54m \text{ km. per sec.}$$

* *Astrophysical Journal*, **54**, 189. See especially Table III.

† If we have two groups each of 144 stars separated by an interval of $2^m\cdot5$ with average velocity 15 km. per sec., the change of velocity per magnitude should be given with a probable error 0·6 km. per sec.

The progression of velocity with magnitude is more than doubled. This result is certainly too high, and it shows indirectly that the spectroscopic parallaxes for this group must be rather more accurate than their authors claim. Reference to the table of probable errors (*Ap. J.*, **53**, 27) shows that the prospect of determining h with greater precision is not very hopeful. It is rather disappointing to find that lack of precision of this constant stands in the way of a more successful determination of the important quantity b. But at least it is desirable to know what is the difficulty to be overcome in arriving at a solution of this problem. An increase in the number of stars observed will not diminish the difficulty. If it is not possible to deduce useful results when the probable error of the absolute magnitudes is taken into account, results obtained by neglecting it must *a fortiori* be worthless; in that case the inclusion of proper-motion data in the investigation can only tend to spoil the results obtainable from radial-velocity data which are not subject to this insidious systematic error.

There is one way in which this difficulty may perhaps be overcome. The progression in mean parallactic velocity with absolute magnitude can be calculated by our method just as well as the progression in cross velocity. But this progression should be zero, and we must therefore assign to the absolute magnitudes that value of the probable error which will make the progression zero. This method of finding h was not thought of until the paper was in type, and we have not had time to test whether it is successful practically.

The Equilibrium of a Rotating Star. By E. A. Milne.

(*Communicated by the Director of the Solar Physics Observatory, Cambridge.*)

I. *Outline of the Investigation.*

1. Professor Eddington's theory * of the interior of a star has been formulated by him so far only for non-rotating stars at rest under the action of no external forces. The present paper is an attempt to extend the theory to masses in rotation. In a later paper it is hoped to take some account of the effect of tidal forces.

2. In his earlier papers Eddington found it possible to obtain a very simple solution of the problem of the radiative equilibrium of a spherical mass of gas by making the assumption that k, the coefficient of absorption of radiation, and $4\pi\epsilon$, the rate of evolution of energy at any point per unit mass, were constant throughout the material. Later he showed that the same solution held good under the less restricted condition that the product of k_r, the coefficient of absorption at distance r from the centre, and $4\pi\bar{\epsilon}_r$, the mean rate of evolution of energy per unit mass

* *M.N.R.A.S.*, 77, 16, 1916; 77, 596, 1917. *Astrophys. Journ.*, **48**, 205, 1918. *Zeits. für Phys.*, 7, 351, 1921.

inside the sphere of radius r, was constant. Making either of these assumptions, it follows that the pressure of radiation is everywhere a constant multiple of the gas pressure, and hence * that ρ/T^3, where ρ is the density and T the temperature, is constant throughout the star; it follows also that the temperature is a linear function of the gravitational potential, and that the temperature gradient at any point is proportional to the value of g. Further, all these results are equivalent to one another.

The simplicity of this solution arises from the circumstance that the differential equation satisfied by the temperature reduces to the same form as that satisfied by the potential—Poisson's equation. The number of separate differential equations which require to be solved is thus reduced to one, namely, Emden's equation †

$$\frac{1}{\xi^2}\frac{d}{d\xi}\left(\xi^2\frac{d\theta}{d\xi}\right) = -\theta^3.$$

But if it is not possible to make this or a similar simplification, the problem reduces in general to the solution of two simultaneous non-linear differential equations, and the mathematical difficulties become serious even in cases of spherical symmetry. For example, if we wished to consider a stellar model consisting of a central core of material generating energy surrounded by a shell of material in which no energy is generated, or even the simpler but more abstract case of a gas-sphere of non-emitting material with a point-source at the centre, then we should have to solve the equations for the case $\epsilon = 0$. This might appear at first sight a simplification, but the equations reduce essentially to a single non-linear equation of the third order which is almost unmanageable.

In particular, the equations for a rotating mass in radiative equilibrium do not admit of the simplification in question—the parallelism between the temperature and the potential is imperfect. The radiation pressure is not a constant multiple of the gas pressure, and thus the distribution of temperature and density is not the same as an adiabatic distribution with $\gamma = \frac{4}{3}$. We have, therefore, to be content with approximate solutions.‡

3. We shall consider approximate solutions in two cases:—

(1) When the angular velocity ω is small; an approximation will be obtained correct to the first power of ω^2;

(2) When the mass is so large that gas pressure is small compared with radiation pressure.§

For simplicity we shall consider only stars in the giant stage, when the material may be treated as a perfect gas; and we shall assume a constant mean molecular weight. To facilitate direct comparison with

* For a star consisting of a perfect gas of constant mean molecular weight. It is only with such stars that we shall be concerned in this paper.

† For the notation, see § 17 below.

‡ The tidal problem is simpler. Here radiation pressure remains a constant multiple of the gas pressure, on Eddington's hypotheses, and the difficulties are purely those arising from the non-spherical form.

§ This case has been considered briefly by Jeans, *M.N.R.A.S.*, 79, 329, 1919. See Section IV. below.

Eddington's work, we shall make the same assumptions as he does concerning the evolution of energy; that is to say, we shall assume in the first instance that k and ϵ are constant, but we shall formulate later the corresponding generalisation.

4. The general character of the state of relative equilibrium as compared with that of a mass at rest can be seen without analysis. If a star is imagined to begin spinning, the equatorial portions will lift to some extent under the action of centrifugal force. Centrifugal force takes over from radiation pressure part of the burden of supporting the peripheral equatorial regions, and hence, if the rate of evolution of heat remains constant a greater total mass can be supported. Thus, for given rate of evolution of energy per unit mass the total mass is greater, or, conversely, for given total mass the luminosity is smaller. The star becomes slightly spheroidal. Since at the equatorial regions part of the mass is supported by centrifugal force, whilst this is not the case at the polar regions, therefore the radiation pressure, and hence the net outward flux of energy, must be smaller at the equator than at the poles. Hence the polar regions will appear hotter than the equatorial regions. Owing to compressibility, the reduction in pressure in the equatorial regions causes a general reduction in pressure throughout the star, with the result that the star as a whole suffers an expansion, which is superimposed on the ellipticity. This increase in area, combined with the smaller rate of evolution of heat per unit mass (for given total mass) gives a reduced effective temperature, averaged over the surface. As compared with a rotating liquid, the ellipticity introduced by a given rotation should be in general smaller, since the central condensation results in a smaller proportion of the mass being situated in regions of large centrifugal force.

5. We adopt polar co-ordinates (r, θ, ϕ) and put $\cos\theta = \mu$, θ being measured from the axis of rotation. The quantity ρ/T^3, though no longer constant, is found to be a function of r only throughout the interior, as far as the first power of ω^2. This may be seen as follows. Since ρ/T^3 is constant when $\omega = 0$, there will exist an approximation of the form

$$\rho/T^3 = \text{const.} + \omega^2 f(r, \mu).$$

Now, if an exact solution could be found, it would be specified by the family of "level surfaces" over which the variables ρ, T, etc., are constant. To the first order, the level surfaces will depart but slightly from spheres, and they will have equations of the form

$$r = \lambda + \omega^2 f_1(\lambda, \mu),$$

where λ is the parameter specifying the surfaces. Since, for this set of values of r, ρ/T^3 must be constant as far as the first power of ω^2, it follows on substitution for r that $f(r, \mu)$ must be independent of μ, to the order required. (Another proof will be given later, depending on potential-theory.) The argument does not apply to ρ and T separately, which are accordingly functions of μ as well as of r. The circumstance that ρ/T^3 is approximately a function of r only is a great help in the solution. It will be shown that ρ/T^3 decreases from the centre outwards.

6. If c is the velocity of light, G the constant of gravitation, and k and ϵ (or k_r and $\bar{\epsilon}_r$, suitably generalised) are as above, then following Eddington we define a quantity β, constant throughout the star, by the equation

$$\beta = 1 - \frac{k\epsilon}{c\mathrm{G}} \qquad \left(= 1 - \frac{k_r\bar{\epsilon}_r}{c\mathrm{G}}\right).$$

As thus defined β is no longer the ratio of the gas pressure to the total pressure, though, of course, it reduces to this when $\omega = 0$. Thus our β must be carefully distinguished from the "β" used by Jeans in the paper already cited. It is still the case that β decreases as the mass increases, but it does not decrease to zero as the mass increases indefinitely; for a given non-zero rotation, it will be shown that as the mass increases the distribution of density must be such that at each point

$$\rho \to \frac{\omega^2}{2\pi\mathrm{G}} \Big/ \left(1 - \frac{k\epsilon}{c\mathrm{G}}\right),$$

so that at each point $\beta \to \omega^2/2\pi\mathrm{G}\rho$.

7. In the approximate solution, the perturbations in the quantities ρ, T, the radius of the star, etc., as compared with the non-rotating case, appear as the sum of two terms, of which one is a function of r only and the other is a function of r multiplied by the Legendre function $\mathrm{P}_2(\mu)$. These may be referred to briefly as the "expansion term" and the "ellipticity term."* The ellipticity terms are found to be proportional simply to ω^2/ρ_m, the expansion terms to $\omega^2/\beta\rho_m$, ρ_m being the mean density of the star. The ellipticity is thus independent of the mass, but the expansion becomes large as the mass increases. For large masses the approximation is indeed invalid, since it holds only on the assumption that $\omega^2/\beta\rho_m$ is small, and we have seen that for large masses β becomes comparable with ω^2/ρ_m; but the result is interesting, as indicating that for large masses a comparatively small rotation has a large effect.

Now Eddington † has pointed out that the actual masses of the stars are clustered about that for which radiation pressure just rises into importance, and has conjectured that the reason for the relative absence of very large masses may be that such masses would be likely to be easily disrupted by small disturbances, since in them radiation pressure counterbalances such a large fraction of gravitation. Our result above is in general confirmation of this. It does not prove that rotation makes a large mass more likely to become unstable than a small one, but it shows a definite effect due to mass. The effect clearly arises from the introduction of radiation pressure. This is important; for Jeans' investigations, both of compressible and incompressible masses, show that in the absence of radiation pressure "the mass of a body has almost no influence on its tendency to break up rotationally." ‡ From the approximate solution of case (2) above it is possible to arrive at

* ρ/T^3 contains no ellipticity term.
† *M.N.R.A.S.*, **77**, 603, 1917.
‡ *Problems of Cosmogony and Stellar Dynamics* (1919), p. 16.

further, but somewhat speculative conclusions on this point, but discussion of them is postponed until § 30.

8. The proportional corrections to the luminosity, etc., due to small rotation are obtained in definite numerical form. Involving only the ratios ω^2/ρ_m and $\omega^2/\beta\rho_m$, they are independent of the mean molecular weight or coefficient of absorption. When the formulæ are applied to a definite case (Eddington's typical giant star), with a suitable value of the rotation, the corrections are found to be fairly small; unless the rotation is so large as to distort the star considerably, the fundamental constants which Eddington derives from his theory are comparatively unaffected. The interest of our results lies chiefly in the qualitative tendencies they indicate.

II. *General Theory. Formulation of the Equations.*

9. *The Equation of Radiative Equilibrium.*—A rigorous treatment of radiative equilibrium necessarily involves integral equations, but, as Eddington, Jeans, and others have shown in certain cases, these can be expressed approximately as differential equations. We first obtain the general equation of radiative equilibrium in Cartesian co-ordinates.

Let B (x, y, z) be the intensity of black-body radiation corresponding to the temperature existing at the point (x, y, z), and let $\mathrm{I}(x, y, z; l, m, n)$ be the intensity of radiation at the point (x, y, z) in the direction specified by the direction-cosines l, m, n. Let ds denote a small element of length in the direction (l, m, n). If k, ρ are the absorption coefficient and density at (x, y, z), the equation of transfer of energy is

$$\frac{1}{k\rho}\frac{d\mathrm{I}}{ds} = \mathrm{B} - \mathrm{I} \quad . \quad . \quad . \quad . \quad . \quad (1)$$

If we put

$$\tau = \int_0^\infty k\rho ds,$$

the integration being performed in the direction (l, m, n) and the suffix o referring to the point (x, y, z), then (1) may be written

$$\frac{d\mathrm{I}}{d\tau} = \mathrm{B} - \mathrm{I} \quad . \quad . \quad . \quad . \quad . \quad (2)$$

Equation (2) can be integrated as it stands, and for all points (x, y, z) which are sufficiently far from the boundary we have

$$\mathrm{I}(x, y, z; l, m, n) = \int_{-\infty}^{0} \mathrm{B}(\tau)e^{+\tau}d\tau = \int_0^\infty \mathrm{B}(-\tau)e^{-\tau}d\tau$$

$$= \int_0^\infty \left(\mathrm{B}_0 - \tau\left(\frac{d\mathrm{B}}{d\tau}\right)_0 + \frac{\tau^2}{2}\left(\frac{d^2\mathrm{B}}{d\tau^2}\right)_0 - \ldots\right)e^{-\tau}d\tau.$$

Integrating term by term, and approximating by retaining only the first four terms, we have

$$\mathrm{I}(x, y, z; l, m, n) = \mathrm{B}_0 - \left(\frac{d\mathrm{B}}{d\tau}\right)_0 + \left(\frac{d^2\mathrm{B}}{d\tau^2}\right)_0 - \left(\frac{d^3\mathrm{B}}{d\tau^3}\right)_0 \quad . \quad (3)$$

We have also

$$\left(\frac{d\mathrm{B}}{d\tau}\right)_0 = \left(\frac{l}{k\rho}\frac{\partial \mathrm{B}}{\partial x} + \frac{m}{k\rho}\frac{\partial \mathrm{B}}{\partial y} + \frac{n}{k\rho}\frac{\partial \mathrm{B}}{\partial z}\right)_0, \quad . \quad . \quad . \quad (4)$$

$$\left(\frac{d^2\mathrm{B}}{d\tau^2}\right)_0 = \left(\frac{l}{k\rho}\frac{\partial}{\partial x} + \frac{m}{k\rho}\frac{\partial}{\partial y} + \frac{n}{k\rho}\frac{\partial}{\partial z}\right)_0^2 \mathrm{B}, \quad . \quad . \quad (5)$$

and so on.

Now suppose that $4\pi\epsilon$ is the energy evolved per unit mass per second at (x, y, z). Then this is equal to the excess of emission over absorption. Consequently, we have

$$4\pi\epsilon\rho = k\rho\iint(\mathrm{B} - \mathrm{I})d\Omega.$$

On inserting for I from (3) and using (4) and (5), we find that the only surviving terms are those arising from the second-order derivatives. Transforming to spherical polar co-ordinates for the direction cosines, we find

$$4\pi\epsilon\rho = -k\rho\int_0^{2\pi} d\phi \int_0^{\pi}\left(\frac{\sin\theta\cos\phi}{k\rho}\frac{\partial}{\partial x} + \frac{\sin\theta\sin\phi}{k\rho}\frac{\partial}{\partial y} + \frac{\cos\theta}{k\rho}\frac{\partial}{\partial z}\right)^2 \mathrm{B}\sin\theta\, d\theta$$

$$= -\frac{4\pi}{3}\left[\frac{\partial}{\partial x}\left(\frac{1}{k\rho}\frac{\partial \mathrm{B}}{\partial x}\right) + \frac{\partial}{\partial y}\left(\frac{1}{k\rho}\frac{\partial \mathrm{B}}{\partial y}\right) + \frac{\partial}{\partial z}\left(\frac{1}{k\rho}\frac{\partial \mathrm{B}}{\partial z}\right)\right].$$

We have finally as the equation of radiative equilibrium

$$\sum_{x,y,z}\frac{\partial}{\partial x}\left(\frac{1}{k\rho}\frac{\partial \mathrm{B}}{\partial x}\right) = -3\epsilon\rho \quad . \quad . \quad . \quad . \quad (6)$$

It is convenient to use E, the energy per unit volume, instead of B. We have

$$\mathrm{E} = a\mathrm{T}^4 = \frac{4\pi}{c}\mathrm{B} = \frac{4\sigma}{c}\mathrm{T}^4,$$

σ being Stefan's constant and c the velocity of light. Equation (6) then becomes

$$\sum_{x,y,z}\frac{\partial}{\partial x}\left(\frac{1}{k\rho}\frac{1}{3}\frac{\partial \mathrm{E}}{\partial x}\right) = -\frac{4\pi\epsilon\rho}{c} \quad . \quad . \quad . \quad (7)$$

10. *The Equations of Mechanical Equilibrium.*—Let the axis of z be taken as the axis of rotation. If p is the gas pressure, ω the angular velocity, V the gravitational potential, the equations of relative equilibrium, taking account of radiation pressure, are

$$\left.\begin{aligned} -\frac{\partial p}{\partial x} - \tfrac{1}{3}\frac{\partial \mathrm{E}}{\partial x} + \rho\frac{\partial \mathrm{V}}{\partial x} + \rho\omega^2 x = 0 \\ -\frac{\partial p}{\partial y} - \tfrac{1}{3}\frac{\partial \mathrm{E}}{\partial y} + \rho\frac{\partial \mathrm{V}}{\partial y} + \rho\omega^2 y = 0 \\ -\frac{\partial p}{\partial z} - \tfrac{1}{3}\frac{\partial \mathrm{E}}{\partial z} + \rho\frac{\partial \mathrm{V}}{\partial z} = 0 \end{aligned}\right\} \quad . \quad . \quad . \quad (8)$$

It follows from these equations that there exists a function of position W given by

$$W = \int \rho d(V + \tfrac{1}{2}\omega^2(x^2 + y^2)), \quad . \quad . \quad . \quad (9)$$

whose partial derivatives are given by

$$\left.\begin{aligned} \frac{\partial W}{\partial x} &= \rho \frac{\partial V}{\partial x} + \rho\omega^2 x \\ \frac{\partial W}{\partial y} &= \rho \frac{\partial V}{\partial y} + \rho\omega^2 y \\ \frac{\partial W}{\partial z} &= \rho \frac{\partial V}{\partial z} \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad (10)$$

Let the constant of integration introduced in the definition of W be chosen so that $W = 0$ at one point of the boundary. Then, since $\rho = 0$ over the boundary, we have also $W = 0$ over the boundary. Clearly we have an integral of equations (8) of the form

$$\begin{aligned} W - p - \tfrac{1}{3}E &= \text{const.} \\ &= 0, \quad . \quad . \quad . \quad . \quad (11) \end{aligned}$$

since W, p, and E are zero over the boundary.* Thus W is the total pressure.

The potential V satisfies Poisson's equation

$$\sum_{x,\,y,\,z} \frac{\partial}{\partial x}\left(\frac{\partial V}{\partial x}\right) = -4\pi G\rho \quad . \quad . \quad . \quad . \quad (12)$$

G being the constant of gravitation. Substituting for $\partial V/\partial x$, etc., from (10) we find

$$\sum \frac{\partial}{\partial x}\left(\frac{1}{\rho}\frac{\partial W}{\partial x}\right) = -4\pi G\rho + 2\omega^2 \quad . \quad . \quad . \quad (13)$$

11. *Parallelism of the Equations.*—In equation (7) k and ϵ can be any functions of position. We will now assume them to be constants.

If $\omega = 0$, equations (7) and (13) are of exactly the same form save for a multiplying constant. Further, E and W are both zero over the boundary. It follows from a theorem in potential theory, that W and E must be proportional throughout the interior, and we have in fact

$$W = \tfrac{1}{3}\frac{Gc}{\epsilon k}E.$$

Equation (11) then shows that p is proportional to E, and Eddington's solution follows immediately. In particular, $p \propto E$ implies $\rho \propto T^3$;

* $E = 0$ is the boundary condition appropriate to the system of equations we are discussing. A rigorous treatment would give the Schwarzschild temperature at the boundary, not a zero temperature. Equation (7) is not in fact valid near the boundary; what we are doing is to solve a fictitious problem (in which our equations are taken as exact), the solution of which practically coincides with the solution of the actual problem except near the boundary. *Cf.* Eddington, *M.N.*, 77, 23, 1916.

hence the temperature equation (7) reduces to Poisson's equation, and T is therefore a linear function of V.

If $\omega \neq 0$, we shall proceed by defining a function U by the relation

$$W = \tfrac{1}{3}\frac{Gc}{\epsilon k}E + \omega^2 U \quad . \quad . \quad . \quad . \quad (14)$$

U will be called the "rotation-pressure function"; it makes its appearance in the rotation problem but not in the tidal problem; it measures the perturbation introduced by rotation.

Inserting for W in (13) in terms of U and using (7), we find that U satisfies the equation

$$\sum_{x,y,z}\frac{\partial}{\partial x}\left(\frac{1}{\rho}\frac{\partial U}{\partial x}\right) = 2 \quad . \quad . \quad . \quad . \quad (15)$$

Further, from (14) U is zero over the boundary. Thus if ρ were known as a function of x, y, z, the solution for U would be unique.

12. We can now assemble the equations embodying the problem. Put

$$\beta = 1 - \frac{k\epsilon}{cG}.$$

Substitute $E = aT^4$ in (7); eliminate W between (11) and (14) and introduce the gas equation $p = (R/m)\rho T$.

Then the problem is to determine the three variables—T, ρ, U—from the three equations

$$\sum_{x,y,z}\frac{\partial}{\partial x}\left(\frac{T^3}{\rho}\frac{\partial T}{\partial x}\right) = -3\pi G\rho(1-\beta) \quad . \quad . \quad . \quad (16)$$

$$\sum_{x,y,z}\frac{\partial}{\partial x}\left(\frac{1}{\rho}\frac{\partial U}{\partial x}\right) = 2 \quad . \quad . \quad . \quad . \quad (17)$$

$$\tfrac{1}{3}aT^4\frac{\beta}{1-\beta} - \frac{R}{m}\rho T + \omega^2 U = 0 \quad . \quad . \quad . \quad (18)$$

with the conditions that T, ρ, U are all zero over the boundary.

13. It is necessary to use these equations in the form appropriate to spherical polar co-ordinates. We measure θ from the axis of rotation, and put $\cos\theta = \mu$. The co-ordinate ϕ can be rejected from symmetry. Equations (16) and (17) then take the form

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{T^3}{\rho}\frac{\partial T}{\partial r}\right) + \frac{1}{r^2}\frac{\partial}{\partial \mu}\left((1-\mu^2)\frac{T^3}{\rho}\frac{\partial T}{\partial \mu}\right) = -3\pi G\rho(1-\beta) \quad . \quad (19)$$

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{1}{\rho}\frac{\partial U}{\partial r}\right) + \frac{1}{r^2}\frac{\partial}{\partial \mu}\left(\frac{1-\mu^2}{\rho}\frac{\partial U}{\partial \mu}\right) = 2 \quad . \quad . \quad (20)$$

14. The equations and conditions so far given are necessary but not sufficient; they do not determine T and ρ uniquely, as we shall see later when we come to find an actual solution. For equations (16), (17), (18) contain no explicit reference to the potential, and are the same whatever the external gravitational field. To remove the indeterminateness in ρ

and T, it is necessary to use the ρ as found to calculate the potential arising from the matter, and then to arrange that this together with any applied external field satisfies the equations of equilibrium.

It will thus be necessary later to have the equations of equilibrium in spherical polars. Equations (10) become

$$\frac{\partial W}{\partial r} = \rho \frac{\partial V}{\partial r} + \rho\omega^2 r(1-\mu^2),$$

$$\frac{\partial W}{\partial \mu} = \rho \frac{\partial V}{\partial \mu} - \rho\omega^2 r^2 \mu.$$

In terms of the Legendre function $P_2(\mu)$ we have

$$1-\mu^2 = \tfrac{2}{3}[1 - P_2(\mu)], \qquad \mu = \tfrac{1}{3}P_2'(\mu).$$

Using these and substituting for W from (14) we have finally

$$\frac{\partial V}{\partial r} = \tfrac{4}{3}a\frac{1}{1-\beta}\frac{T^3}{\rho}\frac{\partial T}{\partial r} + \frac{\omega^2}{\rho}\frac{\partial U}{\partial r} - \tfrac{2}{3}\omega^2 r + \tfrac{2}{3}\omega^2 r P_2(\mu) \quad . \quad (21)$$

$$\frac{\partial V}{\partial \mu} = \tfrac{4}{3}a\frac{1}{1-\beta}\frac{T^3}{\rho}\frac{\partial T}{\partial \mu} + \frac{\omega^2}{\rho}\frac{\partial U}{\partial \mu} + \tfrac{1}{3}\omega^2 r^2 P_2'(\mu). \quad . \quad . \quad (22)$$

15. *The Level Surfaces.*—By analysis similar to that used by Jeans * it can be shown from (10) that ρ is a function of $V + \frac{1}{2}\omega^2(x^2+y^2)$. The same is true of W, and hence from (11) of T. Thus the surfaces $T = \text{const.}$, $\rho = \text{const.}$, $V + \frac{1}{2}\omega^2(x^2+y^2) = \text{const.}$ are the same surfaces, and further $V + \frac{1}{2}\omega^2(x^2+y^2)$ is constant over the boundary. It follows also that the flow of energy is everywhere normal to the level surfaces.

16. We can make use of the existence of level surfaces to replace the condition assumed in §§ 11–14 that ϵ and k must be constant by a more general condition. Assume instead that ϵ and k vary from point to point but are constant over a level surface; this is a reasonable supposition, since the physical conditions are the same at all points on a level surface. Write them k_λ and ϵ_λ, λ being the parameter specifying the surfaces. Integrate both sides of equation (7) throughout the space interior to a level surface and apply Green's theorem. We find

$$\frac{1}{3k_\lambda\rho_\lambda}\int\int\frac{\partial E_\lambda}{\partial n}dS = -\frac{4\pi}{c}\int\int\int\epsilon_\lambda\rho_\lambda\, dx\, dy\, dz$$

$$= -\frac{4\pi\bar{\epsilon}_\lambda}{c}\int\int\int\rho_\lambda dx\, dy\, dz \quad . \quad . \quad (23)$$

where $4\pi\bar{\epsilon}_\lambda$ is the mean rate of evolution of energy per unit mass throughout the material interior to the surface λ.

Now assume that the distribution of the production of energy in the interior is such that

$$k_\lambda\bar{\epsilon}_\lambda = \text{const.},$$

* *Cosmogony*, p. 140. The introduction of radiation pressure makes no difference.

independent of λ; *i.e.* that the coefficient of absorption at any point varies inversely as the mean rate of generation of energy per unit mass interior to the level surface passing through the point. Then applying (23) to the shell between two neighbouring surfaces and again using Green's theorem, it can be shown that at each point E satisfies the equation *

$$\sum_{x,\,y,\,z} \frac{\partial}{\partial x}\left(\frac{1}{\rho}\frac{1}{3}\frac{\partial \mathrm{E}}{\partial x}\right) = -\frac{4\pi k_\lambda \bar{\epsilon}_\lambda \rho}{c} \qquad (24)$$

The quantity β, defined by

$$\beta = 1 - \frac{k_\lambda \bar{\epsilon}_\lambda}{c\mathrm{G}},$$

is now constant throughout the star, and the analysis of §§ 11–14 is applicable without alteration.

This generalisation is the analogue of Eddington's for a star at rest. Recently Eddington † has given theoretical reasons for the condition $k_\lambda \bar{\epsilon}_\lambda = \text{const.}$ being a plausible one.

III. *Approximate Solution for Slow Rotation.*

17. *Solution for no rotation.*—We shall find an approximate solution in the form of small corrections to the case of no rotation, and it is necessary therefore to recover the formulæ for this case as briefly as possible.

From (18) with $\omega = 0$, we have

$$\frac{\mathrm{T}^3}{\rho} = \frac{\mathrm{R}}{m} \cdot \frac{1-\beta}{\frac{1}{3}a\beta} \qquad (25)$$

Inserting for ρ in (19) we have, since the quantities are independent of μ,

$$\frac{1}{r^2}\frac{d}{dr}\left(r^2 \frac{d\mathrm{T}}{dr}\right) = -\mathrm{K}\mathrm{T}^3 \qquad (26)$$

where

$$\mathrm{K} = \tfrac{1}{3}\pi a \mathrm{G}\left(\frac{m}{\mathrm{R}}\right)^2 \frac{\beta^2}{1-\beta}.$$

Introduce dimensionless variables θ and ξ in place of T and r, given by ‡

$$\mathrm{T} = \mathrm{T}_c\theta,$$
$$r = \mathrm{K}^{-\frac{1}{2}}\mathrm{T}_c^{-1}\xi,$$

where T_c is the central temperature. Equation (26) then becomes

$$\frac{1}{\xi^2}\frac{d}{d\xi}\left(\xi^2 \frac{d\theta}{d\xi}\right) = -\theta^3 \qquad (27)$$

We require the solution of this which satisfies

$$\theta(0) = 1, \qquad \theta'(0) = 0.$$

* A rigorous proof appears to be a little difficult.

† *M.N.R.A.S.*, 83, 32, 1922.

‡ This use of θ involves no confusion with the polar co-ordinate θ.

The numerical values of θ and θ' have been given by Emden.* The boundary is given by the root of the equation

$$\theta(\xi) = 0,$$

and if this root is $\xi = \xi_1$ the radius r_1 is given by

$$r_1 = K^{-\frac{1}{2}} T_c^{-1} \xi_1,$$

which is the expression of Lane's law for our case. The value of ξ_1 is 6·9011.

We must also recover Eddington's quartic equation connecting the mass M and β. We have

$$\begin{aligned} M &= 4\pi \int_0^{r_1} \rho r^2 dr \\ &= \frac{4\pi a}{3} \frac{m}{R} \frac{\beta}{1-\beta} T_c^3 \int_0^{\xi_1} K^{-\frac{3}{2}} T_c^{-3} \theta^3 \xi^2 d\xi \quad . \quad . \quad (28) \end{aligned}$$

From the differential equation we find that

$$\int_0^{\xi_1} \theta^3 \xi^2 d\xi = -\xi_1^2 \left(\frac{d\theta}{d\xi}\right)_1 \quad . \quad . \quad . \quad (28A)$$

The value of this can be found from Emden's tables. Squaring (28) we have finally

$$M^2 = \frac{48}{\pi a G^3} \left(\frac{R}{m}\right)^4 \frac{1-\beta}{\beta^4} \xi_1^4 \left(\frac{d\theta}{d\xi}\right)_1^2 \quad . \quad . \quad . \quad (29)$$

which is Eddington's equation. The value of $\xi_1^4 \theta_1'^2$ is 4·060.

18. *Calculation of the Rotation-Pressure Function.*—The particular non-rotating star which we choose to regard as a standard of comparison for the rotating star is to some extent arbitrary. We shall in the first instance compare two stars which have the same central temperature T_c and the same rate of generation of energy per unit mass. Thus β will be the same for the two stars. We build up each star starting from the centre, but it does not follow that the two stars will have the same mass. We shall in fact find that the rotating star has the larger mass. Ultimately, however, we wish to compare stars of the same mass, and hence in the second instance we shall correct our results so as to give stars of equal mass.

It is apparent from (18) that to determine T and ρ as far as the first power of ω^2, the rotation-pressure function U is required only as far as its leading term, the term independent of ω^2. (In a complete solution U would of course depend on ω^2.) Hence in the determination of U from (20) ρ can be given its value in the non-rotating case, which is already known. This value is independent of μ.

A particular solution of (20) independent of μ is obtained by solving the equation

$$\frac{1}{r^2} \frac{d}{dr}\left(\frac{r^2}{\rho} \frac{dU}{dr}\right) = 2 \quad . \quad . \quad . \quad . \quad (30)$$

* *Gaskugeln*, p. 80, Table 7, "$n=3$, $k=\frac{4}{3}$" (Leipzig, 1907).

Calling a solution of this U_1 for the moment, if we write any other solution of (20) in the form

$$U = U_1 + U_2,$$

it follows that U_2 must satisfy the equation

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(\frac{r^2}{\rho}\cdot\frac{\partial U_2}{\partial r}\right)+\frac{1}{r^2}\frac{\partial}{\partial \mu}\left(\frac{1-\mu^2}{\rho}\frac{\partial U_2}{\partial \mu}\right)=0,$$

or, returning to Cartesians,

$$\sum_{x,\,y,\,z}\frac{\partial}{\partial x}\left(\frac{1}{\rho}\frac{\partial U_2}{\partial x}\right)=0 \quad . \quad . \quad . \quad . \quad (31)$$

Suppose now that U_1 is chosen so as to be zero over the boundary. Since U is zero over the boundary, so must U_2 be. But it follows from potential theory that any solution of (31) valid in the interior which is zero over the boundary must be identically zero,* whatever function of position ρ may be. Hence U_2 is identically zero.

We thus see that as far as its leading term the rotation-pressure function is a function of r only, and is given by $U = U_1$. It follows immediately from (18) that to the first order in ω^2, ρ/T^3 is a function of r only. This is the result which was obtained by a different method in § 5 above.

The value of U is now easily obtained. Integrating (30) as it stands, we have

$$\frac{dU}{dr}=\frac{\rho}{r^2}\left(\tfrac{2}{3}r^3+\text{const.}\right).$$

Since U must be finite at the centre, the constant must be zero. Integrating again and remembering that $U = 0$ when $r = r_1$, we have

$$U=-\tfrac{2}{3}\int_r^{r_1}\rho r\,dr \quad . \quad . \quad . \quad . \quad (32)$$

This integral can be evaluated without further quadrature. Substituting for ρ from (25) and returning to the variables θ and ξ we find

$$U=-\frac{2}{3\pi G}\frac{RT_c}{m\beta}\int_\xi^{\xi_1}\theta^3\xi\,d\xi \quad . \quad . \quad . \quad . \quad (33)$$

Now from (27),

$$\begin{aligned}\int_\xi^{\xi_1}\theta^3\xi\,d\xi &= -\int_\xi^{\xi_1}\frac{1}{\xi}\frac{d}{d\xi}\left(\xi^2\frac{d\theta}{d\xi}\right)d\xi\\ &= -\left[\xi\frac{d\theta}{d\xi}\right]_\xi^{\xi_1}-\int_\xi^{\xi_1}\frac{d\theta}{d\xi}d\xi\\ &= \theta+\xi\frac{d\theta}{d\xi}-\xi_1\left(\frac{d\theta}{d\xi}\right). \quad . \quad . \quad . \quad (34)\end{aligned}$$

* Equation (31) is analogous to the equation satisfied by the potential in electrostatics when the dielectric constant is a variable function of position.

We shall denote the last expression by $\phi(\xi)$. It satisfies the differential equation

$$\frac{d\phi}{d\xi} = -\xi\theta^3 \quad . \quad . \quad . \quad . \quad . \quad (35)$$

It will be found * to decrease steadily from the value 1·292 at $\xi = 0$ to zero at $\xi = \xi_1$; at $\xi = \xi_1$ it has a fourth order zero, so that ϕ/θ^4 has a definite limit as $\xi \to \xi_1$, given by

$$\lim_{\xi\to\xi_1}\frac{\phi}{\theta^4} = \lim_{\xi\to\xi_1}\frac{d\phi}{d\xi}\Big/4\theta^3\frac{d\theta}{d\xi} = -\tfrac{1}{4}\xi_1\Big/\left(\frac{d\theta}{d\xi}\right)_1 = 40{\cdot}78 \quad . \quad . \quad (35\text{A})$$

The evaluation of U thus gives

$$\mathrm{U} = -\frac{2}{3\pi\mathrm{G}}\frac{\mathrm{RT}_c}{m\beta}\phi(\xi) \quad . \quad . \quad . \quad . \quad (36)$$

19. *The Variation of* ρ/T^3.—From (18) we have

$$\frac{\rho}{\mathrm{T}^3} = \tfrac{1}{3}a\frac{\beta}{1-\beta}\frac{m}{\mathrm{R}} + \omega^2\frac{m}{\mathrm{R}}\frac{\mathrm{U}}{\mathrm{T}^4}$$

$$= \frac{\tfrac{1}{3}a\beta}{1-\beta}\frac{m}{\mathrm{R}}\left(1 - v\frac{\phi}{\theta^4}\right) \quad . \quad . \quad . \quad . \quad (37)$$

to the first order, where

$$v = \frac{2\omega^2}{\pi a\mathrm{G}}\frac{1-\beta}{\beta^2}\frac{\mathrm{R}}{m\mathrm{T}_c^3} \quad . \quad . \quad . \quad . \quad (38)$$

It is in terms of the small quantity v that the solution henceforth principally appears. A very simple expression can be given for v. For in the non-rotating case we have

$$\rho_c = \frac{\tfrac{1}{3}a\beta}{1-\beta}\frac{m}{\mathrm{R}}\mathrm{T}_c^3,$$

and so to the same order of ω^2,

$$v = \frac{2}{3\pi\mathrm{G}}\frac{\omega^2}{\beta\rho_c}.$$

Further, if ρ_m denotes the mean density, we have (Emden)

$$\frac{\rho_m}{\rho_c} = \int_0^{\xi_1}\theta^3\xi^2 d\xi\Big/\int_0^{\xi_1}\xi^2 d\xi = -\frac{3}{\xi_1}\left(\frac{d\theta}{d\xi}\right)_1 = \frac{1}{54{\cdot}36},$$

and so

$$v = \frac{2}{3\pi\mathrm{G}}\cdot\frac{\omega^2}{\beta\rho_m}\cdot\frac{1}{54{\cdot}36} \quad . \quad . \quad . \quad . \quad (39)$$

We note the appearance of β in the denominator, to which attention has already been drawn in § 7.

The function ϕ/θ^4 increases from 1·292 at the centre to 40·78 at the boundary, the increase being rapid in the outer parts of the star. Thus ρ/T^3 decreases outwards from the centre.

* See Appendix.

20. *Determination of* T *and* ρ *separately.*—We take a new variable Θ defined by

$$T = T_c\Theta \quad . \quad . \quad . \quad . \quad . \quad (40)$$

From (37) we have

$$\rho = \frac{\frac{1}{3}a\beta}{1-\beta}\frac{m}{R}T_c^3\Theta^3\left(1 - v\frac{\phi}{\theta^4}\right) \quad . \quad . \quad . \quad (41)$$

and on inserting for ρ in (19) and changing to the variable ξ we have as the equation for Θ

$$\frac{1}{\xi^2}\frac{\partial}{\partial\xi}\left(\frac{\xi^2}{1 - v\phi/\theta^4}\frac{\partial\Theta}{\partial\xi}\right) + \frac{1}{\xi^2}\frac{\partial}{\partial\mu}\left(\frac{1-\mu^2}{1 - v\phi/\theta^4}\frac{\partial\Theta}{\partial\mu}\right) = -\Theta^3\left(1 - v\frac{\phi}{\theta^4}\right) . \quad (41A)$$

We now put

$$\Theta = \theta + vf \quad . \quad . \quad . \quad . \quad . \quad (42)$$

where f is a function of ξ and μ, and approximate as far as the first power of v. Using the differential equation satisfied by θ and remembering that $\partial\theta/\partial\mu = 0$, we find

$$\frac{1}{\xi^2}\frac{\partial}{\partial\xi}\left[\xi^2\left(\frac{\phi}{\theta^4}\frac{d\theta}{d\xi} + \frac{\partial f}{\partial\xi}\right)\right] + \frac{1}{\xi^2}\frac{\partial}{\partial\mu}\left[(1-\mu^2)\frac{\partial f}{\partial\mu}\right] = -3\theta^2 f + \frac{\phi}{\theta} \quad . \quad (43)$$

Since Θ and θ are both unity for $\xi = 0$, f must be zero for $\xi = 0$, and from symmetry we must have also $\partial f/\partial\xi = 0$ for $\xi = 0$ for all values of μ.

Now assume that f is expansible in a series of Legendre functions

$$f = f_0 + \sum_{n=1}^{\infty} f_n P_n(\mu),$$

the f_n's being functions of ξ only, and such that $f_n(0) = 0$, $f_n'(0) = 0$. Using Legendre's differential equation

$$\frac{\partial}{\partial\mu}\left[(1-\mu^2)\frac{\partial P_n(\mu)}{\partial\mu}\right] = -n(n+1)P_n(\mu),$$

and equating to zero the coefficients of the P_n's in (43), we find

$$\frac{1}{\xi^2}\frac{\partial}{\partial\xi}\left[\xi^2\left(\frac{\phi}{\theta^4}\frac{d\theta}{d\xi} + \frac{df_0}{d\xi}\right)\right] = -3\theta^2 f_0 + \frac{\phi}{\theta} \quad . \quad . \quad (44)$$

$$\frac{1}{\xi^2}\frac{\partial}{\partial\xi}\left[\xi^2\frac{df_n}{d\xi}\right] = f_n\left(\frac{n(n+1)}{\xi^2} - 3\theta^2\right) \quad . \quad . \quad . \quad (45)$$

the latter equation holding for $n \geqslant 1$.

The conditions for $\xi = 0$ are sufficient to determine f_0 uniquely, and the result of solving (44) by quadrature will be given in the Appendix; $f_0(\xi)$ is found to increase steadily from 0 at $\xi = 0$ to 6·00 at $\xi = \xi_1$.

When $n \geqslant 1$, the conditions are not sufficient to fix f_n uniquely; for it is clear that if f_n is a solution of (45), then Af_n is also a solution, where A is an arbitrary constant. For integrals regular at $\xi = 0$ it can

be seen that the expansion of f_n in powers of ξ must commence with the term in ξ^n, and hence to make the solution definite we will select the solution in which the coefficient of ξ^n is unity. We must then write the expression for Θ thus :—

$$\Theta = \theta + v[f_0 + \sum_{n=1}^{\infty} A_n f_n P_n(\mu)] \quad . \quad . \quad . \quad (46)$$

In the sequel the only value of n for which a solution is required is $n=2$, and equation (45) is integrated numerically in this case in the Appendix. The function f_2 increases steadily from 0 at $\xi=0$ to 11·25 at $\xi=\xi_1$.

To determine the A_n's we must first find the potential, as explained in § 14. Substituting for ρ in Poisson's equation, we find

$$\frac{1}{\xi^2}\frac{\partial}{\partial\xi}\left[\xi^2\frac{\partial V}{\partial\xi}\right] + \frac{1}{\xi^2}\frac{\partial}{\partial\mu}\left[(1-\mu^2)\frac{\partial V}{\partial\mu}\right] = -\frac{4\pi G}{KT_c^2}\frac{\frac{1}{3}a\beta}{1-\beta}\frac{m}{R}T_c^3\Theta^3\left(1 - v\frac{\phi}{\theta^4}\right)$$

$$= -\frac{4RT_c}{m\beta}(\theta^3 + 3v\theta^2 f)\left(1 - v\frac{\phi}{\theta^4}\right) \quad . \quad . \quad . \quad (47)$$

In this replace V by

$$\mathbf{V} + v[V_0 + \sum_{n=1}^{\infty} V_n P_n(\mu)],$$

where $\mathbf{V}$ is the potential for the non-rotating star.

Equation (47) then splits up into the three equations,

$$\frac{1}{\xi^2}\frac{d}{d\xi}\left[\xi^2\frac{d\mathbf{V}}{d\xi}\right] = -\frac{4RT_c}{m\beta}\theta^3 \quad . \quad . \quad . \quad (48)$$

$$\frac{1}{\xi^2}\frac{d}{d\xi}\left[\xi^2\frac{dV_0}{d\xi}\right] = -\frac{4RT_c}{m\beta}\left(3\theta^2 f_0 - \frac{\phi}{\theta}\right) \quad . \quad . \quad (49)$$

$$\frac{1}{\xi^2}\frac{d}{d\xi}\left[\xi^2\frac{dV_n}{d\xi}\right] - \frac{n(n+1)}{\xi^2}V_n = -\frac{4RT_c}{m\beta}3\theta^2 A_n f_n \quad . \quad . \quad (50)$$

Using the differential equation for θ, the first can be integrated as it stands. It yields

$$\mathbf{V} = \frac{4RT_c}{m\beta}\theta + \text{const.} \quad . \quad . \quad . \quad . \quad (51)$$

The second can also be integrated, by using the left-hand side of (44). Thus

$$\frac{1}{\xi^2}\frac{d}{d\xi}\left[\xi^2\frac{dV_0}{d\xi}\right] = \frac{4RT_c}{m\beta}\frac{1}{\xi^2}\frac{d}{d\xi}\left[\xi^2\left(\frac{\phi}{\theta^4}\frac{d\theta}{d\xi} + \frac{df_0}{d\xi}\right)\right],$$

giving

$$\frac{dV_0}{d\xi} = \frac{4RT_c}{m\beta}\left[\frac{\phi}{\theta^4}\frac{d\theta}{d\xi} + \frac{df_0}{d\xi}\right] \quad . \quad . \quad . \quad (52)$$

there being no arbitrary constant since V_0 has no singularity at $\xi=0$. Now

$$\int_0^\xi \frac{\phi}{\theta^4}\frac{d\theta}{d\xi}d\xi = -\left[\frac{\phi}{3\theta^3}\right]_0^\xi + \int_0^\xi \frac{1}{3\theta^3}\frac{d\phi}{d\xi}\,d\xi$$

$$= -\frac{\phi}{3\theta^3} - \tfrac{1}{3}\int_0^\xi \xi d\xi$$

$$= -\frac{\phi}{3\theta^3} - \tfrac{1}{6}\xi^2,$$

and hence integrating (52) we have

$$V_0 = \frac{4RT_c}{m\beta}\left[f_0 - \frac{\phi}{3\theta^3} - \tfrac{1}{6}\xi^2\right] + \text{const.} \quad . \quad . \quad . \quad (53)$$

The third equation can be transformed by using (45). We get

$$\frac{1}{\xi^2}\frac{d}{d\xi}\left(\xi^2\frac{dV_n}{d\xi}\right) - \frac{n(n+1)V_n}{\xi^2} = \frac{4RT_c}{m\beta}A_n\left[\frac{1}{\xi^2}\frac{d}{d\xi}\left(\xi^2\frac{df_n}{d\xi}\right) - \frac{n(n+1)f_n}{\xi^2}\right].$$

This shows that a particular solution of (50) is

$$V_n = \frac{4RT_c}{m\beta}A_n f_n,$$

and the general solution will be obtained by adding to this any solution of the same equation deprived of its left-hand member, namely, of

$$\frac{1}{\xi^2}\frac{d}{d\xi}\left(\xi^2\frac{dV_n}{d\xi}\right) - \frac{n(n+1)V_n}{\xi^2} = 0.$$

The solutions of this are known to be ξ^n and $\xi^{-(n+1)}$. Rejecting the latter, since there is no singularity at the centre, and introducing arbitrary constants B_n, we have

$$V_n = \frac{4RT_c}{m\beta}(A_n f_n + B_n\xi^n) \quad . \quad . \quad . \quad (54)$$

Thus finally

$$V = \text{const.} + \frac{4RT_c}{m\beta}\left[\theta + v\left(f_0 - \frac{\phi}{3\theta^3} - \tfrac{1}{6}\xi^2 + \sum_{n=1}^{\infty}(A_n f_n + B_n\xi^n)P_n(\mu)\right)\right]$$

$$= \text{const.} + \frac{4RT_c}{m\beta}\left[\Theta - v\left(\frac{\phi}{3\theta^3} + \tfrac{1}{6}\xi^2\right) + v\sum_{n=1}^{\infty}B_n\xi^n P_n(\mu)\right] \quad . \quad . \quad (55)$$

The next step is to insert our expressions for V, T, ρ, U in the equations of equilibrium (21) and (22). This work is a little tedious, and we omit the details. It is found that to the first order in v (or ω^2) the equations are satisfied if we take

$$B_1 = B_3 = B_4 = \ldots = 0 \quad . \quad . \quad . \quad . \quad (56)$$

and choose B_2 so that

$$\frac{4RT_c}{m\beta}vB_2\xi^2 = \tfrac{1}{3}\omega^2 r^2.$$

This gives

$$B_2 = \tfrac{1}{8}\beta \quad . \quad . \quad . \quad . \quad . \quad (57)$$

It should be observed that the expression $V + \frac{1}{2}\omega^2(x^2+y^2)$, which is the same as $V + \frac{1}{3}\omega^2 r^2[1 - P_2(\mu)]$, is now constant to the first power of ω^2 for all values of r and μ connected by the relation $\Theta = \text{const.}$, *i.e.* is constant over the level surfaces, in accordance with the general theory.

We have still to ensure that V is the actual potential arising from the mass. This will determine the A_n's.

The potential in the free space external to the star must be expressible in the form

$$V_e = \text{const.} + \frac{4RT_c}{m\beta}\left[\frac{C_0}{\xi} + v\sum_{n=1}^{\infty}\frac{C_n}{\xi^{n+1}}P_n(\mu)\right] \quad . \quad . \quad (58)$$

the C_n's being constants, and this and its first derivative must be continuous with the value of V and its derivative at all points on the surface. The surface is given by $\Theta = 0$. We can in fact determine the A's and C's by equating the coefficients of the P_n's in V_e and V and their derivatives over the surface, but the following considerations simplify the analysis.

The surface $\Theta = 0$ will differ from the sphere $\xi = \xi_1$ by terms of the first order in v. Now the density ρ has a zero of the third order at the true boundary, and hence throughout the space between the surfaces $\Theta = 0$ and $\xi = \xi_1$, the density will be of the order v^3. In so far as a first order solution is concerned, then, the density is zero between the true surface and the sphere $\xi = \xi_1$, and hence the potential must arise simply from the distribution of matter inside $\xi = \xi_1$. It will therefore suffice to make the potential and its derivative continuous over the sphere instead of over the surface $\Theta = 0$.

Comparing now the external and internal potentials and their derivatives with respect to ξ at $\xi = \xi_1$, and equating the coefficients of $P_n(\mu)$, we have if $n \neq 2$

$$\frac{C_n}{\xi_1^{n+1}} = A_n f_n(\xi_1),$$

$$-\frac{(n+1)C_n}{\xi_1^{n+2}} = A_n f_n'(\xi_1),$$

which give immediately $C_n = A_n = 0$. On the other hand for $n = 2$, with the value of B_2 given by (57), we have

$$\frac{C_2}{\xi_1^3} = A_2 f_2(\xi_1) + \tfrac{1}{8}\beta\xi_1^2,$$

$$-\frac{3C_2}{\xi_1^4} = A_2 f_2'(\xi_1) + \tfrac{1}{4}\beta\xi_1.$$

These give

$$A_2 = -\tfrac{5}{8}\beta\frac{\xi_1^2}{3f_2(\xi_1) + \xi_1 f_2'(\xi_1)} \quad . \quad . \quad . \quad (59$$

The solution is now definite. The temperature distribution is given by

$$\frac{T}{T_c} = \Theta(\xi) = \theta(\xi) + v\left[f_0(\xi) - \tfrac{5}{8}\beta\frac{\xi_1^2}{3f_2(\xi_1)+\xi_1 f_2'(\xi_1)} f_2(\xi) P_2(\mu)\right] \quad . \quad (60)$$

and the density follows from (37). The numerical value of $\xi_1^2[3f_2(\xi_1)+\xi_1 f_2'(\xi_1)]^{-1}$ is 0·872. (See Appendix.)

21. *Expansion and Ellipticity.*—The boundary is given by $\Theta = 0$, and hence from (60) by

$$\xi = \xi_1 - \frac{v}{\theta_1'}\left[f_0(\xi_1) - \tfrac{5}{8}\beta\frac{\xi_1^2 f_2(\xi_1)}{3f_2(\xi_1)+\xi_1 f_2'(\xi_1)} P_2(\mu)\right] \quad . \quad . \quad (61)$$

This represents an oblate spheroid. Now θ_1' is negative. Thus there is an expansion of the star as a whole, of amount $vf_0(\xi_1)/|\theta_1'|$, and superimposed on this an ellipticity. The expansion is proportional to v, *i.e.* to $\omega^2/\beta\rho_m$, the ellipticity to $v\beta$, *i.e.* to ω^2/ρ_m. Thus the expansion produced by a given rotation depends on the radiation pressure, and increases with it, whilst the ellipticity is independent thereof.

The flux of energy at the surface is proportional to $-\partial T/\partial r$. Now $f_2'(\xi)$ is positive, and $P_2(\mu)$ is positive at the poles and negative at the equator. It follows, therefore, on differentiation of (60), that the flux is greater than the average at the poles and less than the average at the equator. Thus the star will appear hotter at the poles than at the equator. We postpone the determination of the difference until we have made the correction for equal masses.

22. *Mass.*—The mass is given by

$$M = 2\pi\iint \rho r^2 dr d\mu,$$

taken throughout the interior. The ellipticity term in the density contributes nothing to the mass on the average, and we find

$$M = 4\left(\frac{R}{m}\right)^2\left[\frac{1-\beta}{\beta^4}\cdot\frac{3}{\pi a G}\right]^{\frac{1}{2}}\int_0^{\xi_1+\delta\xi_1}\left(1 - v\frac{\phi}{\theta^4}\right)(\theta^3 + 3v\theta^2 f_0)\xi^2 d\xi \quad (62)$$

Here $\delta\xi_1$ denotes the expansion. But as above the density is of a higher order of small quantities than the first in the shell $(\xi_1, \xi_1+\delta\xi_1)$, and the integral in (62) is to our order

$$\int_0^{\xi_1}\left[\theta^3 + v\left(3\theta^2 f_0 - \frac{\phi}{\theta}\right)\right]\xi^2 d\xi.$$

The first term in this has already been evaluated in (28A). For the second, using (44), we have

$$\begin{aligned}\int_0^{\xi_1}\left(3\theta^2 f_0 - \frac{\phi}{\theta}\right)\xi^2 d\xi &= -\int_0^{\xi_1}\frac{d}{d\xi}\left[\xi^2\left(\frac{\phi}{\theta^4}\frac{d\theta}{d\xi} + \frac{df_0}{d\xi}\right)\right]d\xi \\ &= -\xi_1^2\left(\frac{df_0}{d\xi}\right)_1 - \xi_1^2\left(\frac{d\theta}{d\xi}\right)_1 \lim_{\xi\to\xi_1}\frac{\phi}{\theta^4} \\ &= -\xi_1^2 f_0'(\xi_1) + \tfrac{1}{4}\xi_1^3 \quad . \quad . \quad . \quad (62A)\end{aligned}$$

on using (35A). Hence we have

$$M = 4\left(\frac{R}{m}\right)^2\left[\frac{1-\beta}{\beta^4}\cdot\frac{3}{\pi\alpha G}\right]^{\frac{1}{2}}\xi_1^2\,|\,\theta_1'\,|\left[1+v\frac{\frac{1}{4}\xi_1-f_0'(\xi_1)}{|\,\theta_1'\,|}\right] \quad . \quad (63)$$

and this is the modified form of Eddington's quartic equation when there is rotation. The value of the coefficient of v in (63) is found to be about $+6{\cdot}0$ or $+6{\cdot}1$. (See Appendix. It is difficult to compute accurately because $\frac{1}{4}\xi_1$ and $f_0'(\xi_1)$ are of the same sign and nearly of the same magnitude, $|\,\theta_1'\,|$ being small.)

We see that for given β the mass of the star in rotation comes out larger.

23. *The Problem of Rotation when the Mass is given.*—To secure the same mass as in the case of no rotation, the value of ϵ and hence of β must be adjusted. Calling the new values $\epsilon+\delta\epsilon$ and $\beta+\delta\beta$ ($\delta\epsilon$ is negative) we have

$$-k\delta\epsilon = c\mathrm{G}\delta\beta \quad . \quad . \quad . \quad . \quad . \quad (64)$$

In (63) the factor involving β becomes $(1-\beta-\delta\beta)^{\frac{1}{4}}(\beta+\delta\beta)^{-2}$. Now we have

$$\frac{(1-\beta-\delta\beta)^{\frac{1}{4}}}{(\beta+\delta\beta)^2} = \frac{(1-\beta)^{\frac{1}{4}}}{\beta^2}\left[1+\delta\beta\left(-\frac{2}{\beta}-\frac{\frac{1}{2}}{1-\beta}\right)\right].$$

Inserting this in (63), it appears that the mass will be the same as in the non-rotating case provided

$$-\delta\beta\left(\frac{2}{\beta}+\frac{\frac{1}{2}}{1-\beta}\right)+v\frac{\frac{1}{4}\xi_1-f_0'(\xi_1)}{|\,\theta_1'\,|}=0,$$

giving

$$\delta\beta = \frac{2v\beta(1-\beta)}{4-3\beta}\cdot\frac{\frac{1}{4}\xi_1-f_0'(\xi_1)}{\theta_1'\,|} \quad . \quad . \quad (65)$$

It follows, then, from Eddington's formula

$$L = \frac{4\pi c\mathrm{GM}(1-\beta)}{k}$$

for a non-rotating star that if $L+\delta L$ is the luminosity of the rotating star then

$$\frac{\delta L}{L} = -\frac{\delta\beta}{1-\beta} \quad . \quad . \quad . \quad . \quad (66)$$

For future work we require a more detailed expression, thus

$$L+\delta L = \frac{4\pi c\mathrm{GM}}{k}(1-\beta-\delta\beta),$$

and on using (65) with the value of v we find

$$\begin{aligned} L+\delta L &= \frac{4\pi c\mathrm{GM}(1-\beta)}{k}\left[1-\frac{4}{4-3\beta}\cdot\frac{\omega^2}{\pi\mathrm{G}\rho_m}\cdot\frac{\frac{1}{4}\xi_1-f_0'(\xi_1)}{\xi_1}\right] \\ &= \frac{4\pi c\mathrm{GM}(1-\beta)}{k}\left[1-\frac{4}{4-3\beta}\cdot\frac{\omega^2}{\pi\mathrm{G}\rho_m}\times{\cdot}0375\right]. \quad . \quad (66\mathrm{A}) \end{aligned}$$

As evolution proceeds, the moment of momentum of the star remains constant and hence ωr^2 or $\omega^2 r^4$ remains constant. But the mass remaining constant, $r^3\rho_m$ remains constant. Hence $\omega^2/\rho_m \propto r^{-1}$, and so increases as the star contracts. Hence from (66A) the total radiation of a rotating star steadily decreases as the star contracts. (For a non-rotating star the total radiation remains constant.)

24. *External Shape, for given Mass.*—The radius $r_1+\delta r_1$ in any direction is given by

$$r_1+\delta r_1=\left(\frac{3}{\pi a\mathrm{G}}\right)^{\frac{1}{3}}\frac{(1-\beta-\delta\beta)^{\frac{1}{3}}}{\beta+\delta\beta}\frac{\mathrm{R}}{m\mathrm{T}_c}\xi \qquad . \qquad . \quad (67)$$

where ξ has the value given by (61). Now

$$\frac{(1-\beta-\delta\beta)^{\frac{1}{3}}}{\beta+\delta\beta}=\frac{(1-\beta)^{\frac{1}{3}}}{\beta}\left[1-\delta\beta\left(\frac{1}{\beta}+\frac{\frac{1}{2}}{1-\beta}\right)\right].$$

Inserting in (67), we find after some reduction

$$\frac{\delta r_1}{r_1}=\frac{v}{|\theta_1'|}\left[\frac{f_0(\xi_1)}{\xi_1}-\frac{2-\beta}{4-3\beta}\left(\tfrac{1}{4}\xi_1-f_0'(\xi_1)\right)-\beta\frac{\frac{5}{8}\xi_1 f_2(\xi_1)}{3f_2(\xi_1)+\xi_1 f_2'(\xi_1)}\mathrm{P}_2(\mu)\right] \qquad (68)$$

$$=v\left[20{\cdot}56-6{\cdot}1\frac{2-\beta}{4-3\beta}-21{\cdot}00\beta\mathrm{P}_2(\mu)\right]$$

$$=v\left[17{\cdot}5-\frac{3{\cdot}05\beta}{4-3\beta}-21{\cdot}00\beta\mathrm{P}_2(\mu)\right] \qquad . \quad . \quad . \quad . \quad . \quad (69)$$

The expansion term is now not simply proportional to $1/\beta$, but the second term in the square bracket is relatively small.

25. *Distribution of Effective Temperature, for given Mass.*—The net flux of energy at any point of the surface of the star is, to our order,

$$-\frac{16\sigma}{3k}\left[\frac{\mathrm{T}^3}{\rho}\frac{\partial\mathrm{T}}{\partial r}\right]_{r_1+\delta r_1}.$$

We will first evaluate this, ignoring the correction to β. It is equal to

$$-\frac{16\sigma}{3k}\cdot\frac{1-\beta}{\frac{1}{3}a\beta}\cdot\frac{\mathrm{R}}{m}\left(1+v\frac{\phi}{\theta^1}\right)_{\xi_1}\mathrm{K}^{\frac{1}{3}}\mathrm{T}_c{}^2\left(\frac{\partial\Theta}{\partial\xi}\right)_{\xi_1+\delta\xi_1}.$$

This is found to reduce to

$$\frac{16\sigma}{ka}\left(\frac{\pi a(1}{3}\right)^{\frac{1}{3}}(1-\beta)^{\frac{1}{3}}\mathrm{T}_c{}^2|\theta_1'|\left[1+\frac{v}{\theta_1'}\left\{f_0'(\xi_1)-\tfrac{1}{4}\xi_1+\delta\xi_1\theta_1''-\beta\frac{\frac{5}{8}\xi_1{}^2 f_2'(\xi_1)}{\ldots}\mathrm{P}_2(\mu)\right\}\right].$$

From Emden's differential equation the value of θ_1''/θ_1' is found to be $-2/\xi_1$, and the value of $\delta\xi_1$ is given by (61). The square bracket in the last expression then becomes

$$1+\frac{v}{|\theta_1'|}\left\{\tfrac{1}{4}\xi_1-f_0'(\xi_1)-\frac{2}{\xi_1}f_0(\xi_1)+\beta\left(f_2'(\xi_1)+\frac{2}{\xi_1}f_2(\xi_1)\right)\frac{\frac{5}{8}\xi_1{}^2\mathrm{P}_2(\mu)}{3f_2(\xi_1)+\xi_1 f_2'(\xi_1)}\right\}.$$

To correct to the given mass, the factor $(1-\beta)^{\frac{1}{2}}$ must be replaced by $(1-\beta-\delta\beta)^{\frac{1}{2}}$. If now the effective temperature at any point is $T_1+\delta T_1$, T_1 being the effective temperature of the non-rotating star, the expression we have arrived at is equal to $\sigma(T_1+\delta T_1)^4$. After some tedious reduction the final result is

$$\frac{\delta T_1}{T_1}=\frac{v}{|\theta_1'|}\left[-\frac{f_0(\xi_1)}{2\xi_1}+\frac{1-\beta}{4-3\beta}\left(\tfrac{1}{4}\xi_1-f_0'(\xi_1)\right)\right.$$
$$\left.+\tfrac{5}{3\cdot 2}\beta\xi_1\frac{2f_2(\xi_1)+\xi_1 f_2'(\xi_1)}{3f_2(\xi_1)+\xi_1 f_2'(\xi_1)}P_2(\mu)\right] \quad . \quad (70)$$

$$=v\left[-10{\cdot}28+6{\cdot}1\frac{1-\beta}{4-3\beta}+20{\cdot}3\beta P_2(\mu)\right]$$

$$=v\left[-8{\cdot}76-\frac{1{\cdot}5\beta}{4-3\beta}+20{\cdot}3\beta P_2(\mu)\right] \quad . \quad . \quad . \quad (71)$$

The negative sign in the expansion term indicates that the effective temperature is smaller, averaged over the surface, than for the star at rest. The positive sign in the ellipticity term indicates that the poles appear hotter than the equator. It should be observed that the ellipticity term in the effective temperature is relatively more important than that in the radius.

26. *Range of Validity of the Foregoing Approximation.* — The approximation will hold provided v is sufficiently small, but it is desirable to give a more precise meaning to the latter expression. Now the first order solution will have inappreciable errors so long as the product of v and any numerical coefficient it is associated with is fairly small compared with unity. A glance at the details of the analysis will show that errors will arise first through the term $1-v\phi/\theta^4$, which occurs in the expression for ρ: first, because this term occurs in the denominator in equation (41A); secondly, because the value of ϕ/θ^4 at the boundary, namely 40·78, is the largest coefficient * that is associated with v. We shall therefore be safe in taking the errors of our approximation to be negligible, provided $v\phi/\theta^4$ does not exceed, say, $\frac{1}{5}$.

To make the meaning of this condition more intelligible, let n be the ratio of centrifugal force at the equator to the force of gravity (calculated as though the star remained a sphere). Then

$$n=\frac{\omega^2 r}{GMr^{-2}}=\frac{\omega^2}{\frac{4}{3}\pi G\rho_m} \quad . \quad . \quad . \quad . \quad (72)$$

From (39) and the preceding equation, we have

$$v=\frac{2\omega^2}{3\pi G\rho_m\beta}\cdot\frac{3|\theta_1'|}{\xi_1} \quad . \quad . \quad . \quad (73)$$

* Except in § 25, where in the expression for $(T_1+\delta T_1)^4$ (not given), from which (70) is deduced, v occurs with the coefficient $81{\cdot}2\beta P_2(\mu)$. But this is of secondary importance.

whilst since the value of ϕ/θ^4 at the boundary is $\frac{1}{4}\xi_1/|\theta_1'|$, the condition concerning the size of v gives

$$v\frac{\frac{1}{4}\xi_1}{|\theta_1'|}\leqslant\tfrac{1}{5} \quad . \quad . \quad . \quad . \quad . \quad (74)$$

Combining (72), (73), and (74), we find at once that the condition becomes

$$n\leqslant 0{\cdot}3\beta \quad . \quad . \quad . \quad . \quad . \quad (75)$$

For example, for stars of small mass for which β is close to unity, our approximations hold up to periods of rotation so short that centrifugal force is 0·3 times gravity. For more massive stars the biggest permissible angular velocity decreases, tending to zero as the mass increases indefinitely. We again encounter the point, already referred to, that for large masses the character of the deviation from the non-rotating case is entirely different.

As an example, for Eddington's typical giant star of density ·002, for which $\beta=0{\cdot}826$, the smallest permissible period given by (75) is 6·2 days.

27. *Application to an Actual Case.*—We shall take Eddington's typical star* just mentioned, and assume a rotation period of seven days. The star has a radius of 7×10^6 kms., an effective temperature of 6500° (type F 7), a mean molecular weight of 2·8, and its mass is $1{\cdot}5\odot$. But for our purpose we require only the mean density and the value of β.

Using the formulæ established, we find that

$$v=0{\cdot}00382, \qquad \delta\beta=0{\cdot}0044.$$

It is then found that the total luminosity is 2·5 per cent. less than for no rotation. We find further

$$\frac{\delta r_1}{r_1}= \quad {\cdot}0606-{\cdot}0664\mathrm{P}_2(\mu),$$

$$\frac{\delta \mathrm{T}_1}{\mathrm{T}_1}=-{\cdot}0366+{\cdot}0640\mathrm{P}_2(\mu).$$

Thus there is an expansion of the boundary as a whole, of an amount equal to 6·1 per cent. of the radius, and superimposed on this a contraction of the polar diameter of 6·6 per cent. and an expansion of the equatorial diameter of 3·3 per cent. (since $\mathrm{P}_2(1)=1$ and $\mathrm{P}_2(0)=-\frac{1}{2}$). The equatorial axis is thus about 10 per cent. longer than the polar axis. Also there is a reduction in effective temperature, averaged over the whole surface, of 3·7 per cent., and superimposed on this an increase of temperature at the poles of 6·4 per cent. and a decrease of temperature at the equator of 3·2 per cent. The ratio of centrifugal force to gravity at the equator is about 0·19.

Eddington deduces the value of the absorption coefficient k from the formula

$$k=\frac{4\pi c\mathrm{GM}(1-\beta)}{\mathrm{L}} \quad . \quad . \quad . \quad . \quad (76)$$

* *Zeit. für Phys., loc. cit.*, p. 375.

the star being assumed to be at rest. On the basis of the theory of the present paper, if the star is in fact rotating its observed luminosity L will be smaller than the value for the star at rest, and consequently if the observed luminosity is used in (76) the value of k arrived at will be too large—in the case of the typical star 2·5 per cent. too large. This error, however, is negligible compared with other uncertainties entering into the computed value of k, such as the mean luminosity of stars of given mass and the mean molecular weight. Thus we may say that up to a relative elongation of the equatorial axis of 10 per cent., the effect of rotation on the deduced value of k is negligible.

IV. *Approximate Solution for Large Masses.*

28. It is known that in a body of very great mass in radiative equilibrium the pressure of radiation must preponderate over the gas pressure. Let us then to a first approximation neglect the gas pressure. Then equation (11) gives

$$W = \tfrac{1}{3}E \quad . \quad . \quad . \quad . \quad . \quad (76)$$

Equations (7) and (13) are valid under all circumstances, without restriction on k or ϵ. We repeat them for convenience,

$$\sum_{x,y,z} \frac{\partial}{\partial x}\left(\frac{1}{k\rho}\frac{1}{3}\frac{\partial E}{\partial x}\right) = -\frac{4\pi\epsilon\rho}{c} \quad . \quad . \quad . \quad . \quad (77)$$

$$\sum_{x,y,z} \frac{\partial}{\partial x}\left(\frac{1}{\rho}\frac{\partial W}{\partial x}\right) = -4\pi G\rho + 2\omega^2 \quad . \quad . \quad . \quad (78)$$

Now insert (76) in (78) and compare with (77). We see that at each point we must have

$$-\frac{4\pi k\epsilon\rho}{c} = -4\pi G\rho + 2\omega^2,$$

provided only k is constant or varies comparatively slowly from point to point. This gives

$$\frac{\omega^2}{2\pi G\rho} = 1 - \frac{k\epsilon}{cG} \quad . \quad . \quad . \quad . \quad . \quad (79)$$

This result may be stated in words as follows:—

> *In a rotating mass which is so large that the gas pressure is negligible compared with the pressure of radiation, the material must arrange itself so that the specific volume is a certain linear function of the rate of evolution of energy per unit mass.*

Relation (79) shows that ρ is greatest where ϵ is greatest, so that the distribution of density is a plausible one, since ϵ may be presumed to increase towards the interior. The result is of course a limiting result—it states the law towards which the distribution of density must tend in a rotating mass as the mass increases. The result is not restricted to small angular velocities. It fails in fact only when $\omega = 0$.

29. The details of any particular solution can only be worked out when definite assumptions are made about ϵ. But one general result can be obtained—the luminosity is given immediately on integration of (79). We have

$$L = \iiint 4\pi\rho\epsilon dxdydz = \iiint \frac{4\pi cG}{k}\left(1 - \frac{\omega^2}{2\pi G\rho}\right)\rho dxdydz$$

$$= \frac{4\pi cGM}{k}\left(1 - \frac{\omega^2}{2\pi G\rho_m}\right) \quad . \quad . \quad . \quad (80)$$

This result has been obtained by a slightly different method by Jeans.* Its discussion is postponed to § 32 below.

30. *A Particular Model.*—The model considered in Section III. was that for which ϵ (or $k\epsilon$) is constant. When the mass is so large that ordinary pressure can be neglected, (79) shows that if ϵ (or $k\epsilon$) is constant, then ρ is constant. *In a sufficiently massive rotating star in which the energy is evolved in accordance with Eddington's assumptions, the density is constant throughout the material.* It should be observed that this again is a limiting result. It is valid provided $\omega \neq 0$.

But if the density is constant, the material cannot be distinguished from a liquid (from the point of view of its equilibrium). Therefore, the boundary conditions being the same, *it must take up one of the forms of a rotating gravitating liquid.* All such forms are possible forms; the only condition is that the mass must be sufficiently large.

Thus, if Eddington's assumptions concerning the distribution of the internal evolution of energy correspond in any way with the conditions actually ruling in the interior of a star, the forms of rotating liquids would seem to assume a new importance in cosmogony. Presumably the extensive investigations of the stability and methods of break-up of rotating liquids apply as they stand to the massive gaseous model we are considering. Mass by itself, to quote Jeans again, has little or no effect on the tendency of a rotating liquid to instability; but it is only when the mass is sufficiently large that the dynamics of a compressible mass resembles that of an incompressible one and thus that break-up by fission becomes a much more likely contingency than Roche's equatorial break-up. Jeans' criterion† as to which of these methods of break-up occurs first in his compressible (adiabatic) model depends on the value of γ, the ratio of the specific heats. But in our model, in which radiation pressure and the distribution of the internal evolution of energy are taken into account, fissional break-up should always precede equatorial break-up if the mass is sufficiently large. In the rotation problem equatorial break-up results in the ejection of matter from the equator, whilst fissional break-up results in a division into two unequal but comparable masses. Hence our model, by repeated fissional disruption, should lead finally to a number of bodies of comparable mass.

The model we have been considering is a highly idealised one. Moreover even for it our results as stated will not apply rigorously, since the solution cannot hold near the boundary, and therefore parts,

* *M.N.R.A.S.*, **79**, 330 (1919).
† *Cosmogony*, p. 185, etc.

at least, of the stability investigations cannot be strictly applicable. But the general tenour of our results is to show that for very large masses compressibility is of secondary importance; when radiation pressure is very large, the form of equilibrium cannot depend very much on the equation of state of the material but is determined chiefly by the distribution of energy-production in the interior.

§ 31. To sketch briefly the details of the model just discussed, we have to a first approximation (since the density is constant to a first approximation)

$$\tfrac{1}{3}aT^4 = W = \rho[V + \tfrac{1}{2}\omega^2(x^2+y^2)] - \rho[V + \tfrac{1}{2}\omega^2(x^2+y^2)]_1 \quad . \quad (81)$$

the suffix 1 referring to the boundary. Thus when the particular rotating form is selected the temperature distribution is known.

If β has its usual meaning, the value of the constant density is given by (79) as

$$\rho = \frac{\omega^2}{2\pi G\beta} \quad . \quad . \quad . \quad . \quad . \quad (82)$$

To obtain a second approximation to the density, we can employ this value in the equation of state, giving

$$W - \tfrac{1}{3}E - \frac{RT}{m} \cdot \frac{\omega^2}{2\pi G\beta} = 0 \quad . \quad . \quad . \quad (83)$$

If now we assume as a second approximation

$$\rho = \frac{\omega^2}{2\pi G\beta}(1+\rho_1) \quad . \quad . \quad . \quad . \quad (84)$$

we have from equations (77) and (78)

$$\sum_{x,\,y,\,z} \frac{\partial}{\partial x}\left(\frac{1}{\rho}\frac{\partial(W - \frac{1}{3}E)}{\partial x}\right) = -4\pi G\rho\beta + 2\omega^2$$
$$= -2\omega^2\rho_1,$$

and then using (83) we have to the first order

$$\frac{R}{m}\nabla^2 T = -2\omega^2\rho_1 \quad . \quad . \quad . \quad . \quad (85)$$

This determines the correction to the density from the temperature distribution, which is already known from (81).

It is desirable to give a direct verification that for this particular model the phrase "sufficiently large" as applied to the mass means "large compared with the sun." The condition that radiation pressure should be predominant is that $(R/m)\rho T$ should be small compared with $\frac{1}{3}aT^4$. Using the value of T given by (81) this means that

$$(R/m)\rho^{\frac{1}{4}}(\tfrac{1}{3}a)^{-\frac{1}{4}}[\{V + \tfrac{1}{2}\omega^2(x^2+y^2)\} - \{\ldots\}_1]^{-\frac{1}{4}}$$

must be small compared with unity. Since ρ is constant the condition is obviously not satisfied near the boundary, but we will ignore this and test it at the centre. The value of V centre – V surf. will be compar-

able (as regards order of magnitude) with its value for a homogeneous sphere of density ρ, which is $\frac{2}{3}\pi G\rho r^2$, and the test expression is then

$$(R/m)\rho^{\frac{1}{3}}(\tfrac{1}{3}a)^{-\frac{1}{3}}(\tfrac{2}{3}\pi G\rho r^2)^{-\frac{1}{3}}(1-3\omega^2/4\pi G\rho)^{-\frac{2}{3}}$$
$$=\frac{R}{m}\left(\frac{\pi aG^3}{18}\right)^{-\frac{1}{3}}M^{-\frac{1}{3}}\left(1-\frac{3\omega^2}{4\pi G\rho}\right)^{-\frac{2}{3}}.$$

It follows that the condition will be satisfied provided $M/\odot$ is large compared with

$$\left(\frac{R}{m}\right)^2\left(\frac{18}{\pi aG^3}\right)^{\frac{1}{2}}\left(1-\frac{3\omega^2}{4\pi G\rho}\right)^{-\frac{3}{2}}/\odot$$
$$=1{\cdot}3\left(1-\frac{3\omega^2}{4\pi G\rho}\right)^{-\frac{3}{2}},$$

taking $m=2$. Thus the mass must be large compared with a mass somewhat larger than that of the sun.

32. *Comparison with Results of Section III.*—Formula (80) gives the luminosity of a massive rotating star for any angular velocity, and formula (66A) gives the luminosity of a rotating star for small angular velocities without (so far) any restriction on the mass. They are of the same form but the second does not transform into the first as $\beta\to 0$, though it might perhaps be expected to do so, β tending to zero for massive non-rotating stars. The reason is that when β becomes small the analysis of Section III. which leads to (66A) is invalid, since v becomes large. Thus the results of Section III. are not valid for large masses.

For a massive rotating star of the model considered in Section III., equation (79) shows that

$$\beta\to\frac{\omega^2}{2\pi G\rho_m} \quad . \quad . \quad . \quad . \quad . \quad (86)$$

(since $\rho=\text{const.}=\rho_m$ in the limit). The value of β to be employed in (66A) is, of course, the value of β for the corresponding non-rotating star of the same mass, but if we use the value given by (86), formulæ (66A) and (80) are in approximate agreement. It is easy to see, however, that even with the value of β given by (86) the analysis of Section III. is invalid. For it gives

$$v=\frac{2}{3\pi G}\cdot\frac{\omega^2}{\beta\rho_m}\cdot\frac{3|\theta_1'|}{\xi_1}=\frac{4|\theta_1'|}{\xi_1},$$

and hence the value of $v\phi/\theta^4$ at the boundary is unity. This violates the condition given in § 26.

It is instructive to notice that the diminution in luminosity caused by rotation is much larger when the mass is large (80) than when it is small (66A). This is in accordance with the general trend of our investigation, namely, that the effect of rotation is more important for large masses.

V. *Summary.*

33. The general equations of equilibrium of a rotating gaseous mass in radiative equilibrium are formulated, taking account of radiation

pressure and the internal generation of energy. Adopting Eddington's assumptions as to the generation of energy, namely, that the product of the absorption coefficient at any point and of the mean rate of generation per unit mass inside that point is constant, it is found to be no longer true (as for no rotation) that the pressure of radiation is a constant multiple of the gas pressure, or that the gas arranges itself as though it were in adiabatic equilibrium with $\gamma=\frac{4}{3}$. Hence ρ/T^3 is not constant throughout the interior.

By the introduction of a "rotation-pressure" function, it is shown that for a star rotating with a small angular velocity ρ/T^3 decreases steadily from the centre outwards, but is a function of r only, not of the angle θ (using spherical polar co-ordinates). From this an approximate solution for a slowly rotating giant star is deduced. As compared with a star of the same mass and central temperature at rest, the luminosity of a rotating star should be smaller, its mean radius larger and its mean effective temperature smaller. The effective temperature should be greater near the poles than near the equator. Simple physical explanations of these results are given. The luminosity (total radiation) of a rotating star should decrease with advancing evolution. Numerical applications are given in § 27.

The corrections due to rotation appear as the sum of two terms, of which one is the harmonic term $P_2(\cos\theta)$ and has a coefficient ω^2/ρ_m, the other is independent of θ and has a coefficient $\omega^2/\beta\rho_m$. The latter term becomes relatively large for large masses, and it is shown that owing to radiation pressure the effect of rotation is in general more important for large masses than for small ones, in general support of Eddington's conjecture as to the origin of the observed order of magnitude of stellar masses.

A separate investigation is given for masses large compared with the sun, for which gas pressure is negligible compared with radiation pressure. The state of such masses depends almost entirely on the distribution of energy-production in the interior. It is shown that the specific volume at any point must be a certain linear function of $k\epsilon$. For a massive rotating star on Eddington's model the form taken up in the limit is that of a rotating gravitating liquid of constant density. Some speculations as to the bearing of these results on cosmogony are given.

VI. *Appendix. Integration of the Differential Equations.*

34. Equation (44) when expanded can be reduced by means of Emden's differential equation to the form

$$\frac{d^2f_0}{d\xi^2}+\frac{2}{\xi}\frac{df_0}{d\xi}+3\theta^2f_0=\frac{2\phi}{\theta}-\frac{d\theta}{d\xi}\frac{d}{d\xi}\left(\frac{\phi}{\theta^4}\right)$$
$$=F(\xi), \quad . \quad . \quad . \quad . \quad (87)$$

say. There is no difficulty in integrating this equation numerically (with $f_0(0)=0, f_0'(0)=0$) once F has been calculated, but the determination of F involves considerable labour. This is owing to the fourth

order zero in ϕ at $\xi = \xi_1$, which makes the direct calculation of ϕ/θ^4 from Emden's tables (and still more of its derivative) impossible in the neighbourhood of $\xi = \xi_1$. Emden's tabulated values of θ and θ' are not sufficiently accurate, nor are they given to enough significant figures, for ϕ/θ^4 to be determined after $\xi = 5{\cdot}0$ or its derivative after $\xi = 4{\cdot}0$ even to one significant figure. We require them up to $\xi = \xi_1 = 6{\cdot}9011$.

For the smaller values of ξ, we can use the direct formula

$$\frac{d}{d\xi}\left(\frac{\phi}{\theta^4}\right) = -\frac{\xi}{\theta}\left(1 + \frac{4\phi}{\theta^4} \cdot \frac{1}{\xi}\frac{d\theta}{d\xi}\right).$$

For the larger values recourse must be had to a power series. This is found to be

$$\frac{\phi}{\theta^4} = -\tfrac{1}{4}\xi_1\left(\frac{d\xi}{d\theta}\right)_1\left[1 + \frac{12}{5}\frac{\eta}{\xi_1} + \frac{8}{5}\frac{\eta^2}{\xi_1^2} + \frac{4}{35}\frac{\eta^3}{\xi_1^3} + \left(\frac{3}{70} - \tfrac{1}{8}\xi_1^4\theta_1'^2\right)\frac{\eta^4}{\xi_1^4} + \ldots\right],$$

where

$$\eta = \xi - \xi_1.$$

The values of F, together with other useful functions partly re-calculated from power series and partly deduced direct from Emden (corrected) are given in Table I.

35. Near $\xi = 0$ the following expansion in series was obtained for f_0,

$$f_0 = 0{\cdot}43066\xi^2 - 0{\cdot}081022\xi^4 + 0{\cdot}017154\xi^6.$$

Equation (87) was then integrated numerically, starting with this expansion and continuing with a crude "small-arcs" method, with the intervals for which the functions are tabulated in Table I. The resulting values of $f_0(\xi)$ and $f_0'(\xi)$ are given in Table II. The third decimal place has been left in but is quite unreliable.

TABLE I.

ξ.	θ.	$-\theta'$.	ϕ.	ϕ/θ.	ϕ/θ^4.	F.
0	1	0	1·2920	1·2920	1·2920	2·584
0·25	·98968	·08179	1·2612	1·2743	1·3142	2·564
0·50	·95987	·15495	1·1744	1·2235	1·3835	2·505
0·75	·91355	·21270	1·0460	1·1450	1·5018	2·413
1·00	·85505	·25219	·8948	1·0465	1·6742	2·296
1·25	·78897	·27370	·7388	·9364	1·9068	2·164
1·50	·71948	·27993	·5926	·8236	2·2114	2·027
1·75	·64996	·27460	·4614	·7099	2·5853	1·880
2·00	·58282	·26149	·3518	·6037	3·0493	1·741
2·50	·46109	·22396	·1932	·4190	4·2739	1·483
3·00	·35921	·18393	·0994	·2768	5·971	1·266
3·50	·27629	·14859	·0482	·1745	8·272	1·111
4·00	·20942	·11998	·0215	·1026	11·175	·986
4·50	·15529	·09748	·00862	·0555	14·82	·872
5·00	·11110	·08003	·00280	·0252	18·78	·779
6·00	·04384	·05599	$1{\cdot}07 \times 10^{-4}$	$2{\cdot}45 \times 10^{-3}$	29·11	·661
6·80	·00434	·04360	$1{\cdot}398 \times 10^{-8}$	$3{\cdot}22 \times 10^{-6}$	39·36	·606
6·9011	0	·04231	0	0	40·78	·600

TABLE II.

ξ.	f_0.	f_0'.	ξ.	f_0.	f_0'.
0	0	0	2·50	1·384	·699
0·25	·027	·210	3·00	1·742	·735
0·50	·103	·393	3·50	2·126	·802
0·75	·218	·530	4·00	2·550	·895
1·00	·362	·619	4·50	3·023	·998
1·25	·523	·667	5·00	3·548	1·102
1·50	·692	·687	6·00	4·752	1·305
1·75	·864	·691	6·80	5·856	1·455
2·00	1·037	·690	6·9011	6·003	1·473

The relation

$$\int_0^{\xi_1}\left(3\theta^2 f_0 - \frac{\phi}{\theta}\right)\xi^2 d\xi = \xi_1^2\left[\tfrac{1}{4}\xi_1 - \left(\frac{df_0}{d\xi}\right)_1\right] \quad . \quad . \quad (88)$$

(proved as (62A) above) may be used to check the accuracy of the solution. From the tables,

$$\xi_1^2[\tfrac{1}{4}\xi_1 - f_0'(\xi_1)] = 6{\cdot}9011^2[1{\cdot}725 - 1{\cdot}473] = 12{\cdot}00,$$

whilst integration of the left-hand side of (88) by Simpson's rule, using the values of f_0 from Table II., gave 12·37. The agreement is sufficient for our purpose.* The value of the coefficient in the correcting term to Eddington's quartic equation is

$$\frac{\tfrac{1}{4}\xi_1 - f_0'(\xi_1)}{|\theta_1'|},$$

and from Table I. and Table II. this is found to be 5·95. But if we derive it from the result of integrating the left-hand side of (88), the value is found to be 6·15. The latter value is probably the more accurate, and consequently the value 6·1 has been adopted in the text.

36. The second equation which has to be integrated numerically is (45) with $n = 2$, which written out is

$$\frac{d^2 f_2}{d\xi^2} + \frac{2}{\xi}\frac{df_2}{d\xi} = f_2\left(\frac{6}{\xi^2} - 3\theta^2\right).$$

TABLE III.

ξ.	f_2.	f_2'.	ξ.	f_2.	f_2'.
0	0	0	2·50	2·802	1·206
0·25	·062	·487	3·00	3·410	1·225
0·50	·237	·901	3·50	4·050	1·333
0·75	·499	1·192	4·00	4·761	1·508
1·00	·817	1·354	4·50	5·570	1·729
1·25	1·163	1·409	5·00	6·459	1·980
1·50	1·513	1·392	6·00	8·746	2·526
1·75	1·854	1·339	6·80	10·945	2·971
2·00	2·181	1·279	6·9011	11·248	3·022

* For exact agreement we should have to have $f_0'(\xi_1) = 1{\cdot}465$ instead of 1·473 as found.

The expansion near $\xi = 0$ of the particular solution required in the text is found to be

$$f_2 = \xi^2 - \tfrac{3}{14}\xi^4 + \cdot 045635\xi^6 - \cdot 008856\xi^8,$$

and using this expansion initially and continuing by the "small-arcs" method the solution was calculated and is given in Table III.

As a check, we should have

$$\int_0^{\xi_1} \left(\frac{6}{\xi^2} - 3\theta^2\right) f_2 \xi^2 d\xi = \xi_1^2 f_2'(\xi_1).$$

From the table the right-hand side is 143·9, whilst by integration by Simpson's rule the left-hand side was found to be 144·6. This agreement is amply sufficient.

The accuracy of the solutions of both equations depends on the accuracy of Emden's values for ξ_1 and θ_1', which have been adopted throughout. Where necessary others of Emden's figures have been corrected so as to be consistent with these.

The Galactic Distribution of the Spiral Nebulæ, with Special Reference to Galactic Longitude. By J. H. Reynolds. (Plates 4, 5, 6.)

In 1910 Hinks collected all the information then available as to the spiral nebulæ and globular clusters, and his papers in *M.N.*, **71**, together with Hardcastle's paper in *M.N.*, **74**, give an account of the galactic distribution of these and other classes of nebulæ. The principal results of his work were the refutation of the commonly accepted statement that there is a concentration of spirals towards each galactic pole dependent on latitude only, and the establishment of the fact that the globular clusters lie for the most part in one hemisphere of the sky, to the pole of which he assigned the position of galactic longitude 300°, on the galactic equator.

In his twelfth paper on "Colours and Magnitudes in Stellar Clusters," Shapley discusses the question of the situation of the solar system in relation to the galactic plane, and he adduces valid reasons for the conclusion that this position is not nearly central as hitherto supposed, but considerably excentric, so that the limit of the Galaxy is three times greater in longitude 325° than in the opposite direction. The apex in longitude 325° corresponds to the star clouds in Sagittarius, and the antapex in 145° lies in Auriga. If we consider this apex and the antapex as the poles of a great circle, then this circle corresponds to the galactic longitudes 55° and 235°. Shapley also deals with the globular clusters concentrated in the southern hemisphere of this sphere, basing his conclusion on Melotte's examination of the globular clusters on the Franklin-Adams plates. The question naturally arises as to whether the spiral nebulæ show any systematic arrangement with respect

to the sphere thus defined, and if reference is made to the diagram accompanying a paper by the writer on "The Galactic Distribution of the Large Spiral Nebulæ," it will be seen that a very large proportion of the spirals 10′ and over in angular diameter are concentrated in the northern hemisphere of the sphere. This concentration is mentioned in the text as follows:—"The distribution shown on the diagram is in accordance with Hinks's results that, whatever the law of distribution may be, it must be regarded as a function of both longitude and latitude; in fact, longitude seems the more important of the two. The hemisphere bounded by 50° to 230° contains nineteen examples, whereas the hemisphere 230° to 50° (through 360°) contains only four, two of which are very near the south galactic pole." The correspondence between this division of the sphere and that of Shapley is obvious, and evidently has an important bearing on the question of the relationship between the spirals and the galaxy, which has been made the subject of recent papers by the writer.

The conclusion that the globular clusters are, with few exceptions, confined to the southern hemisphere of the sphere, while the larger spirals are almost all congregated in the northern, is an interesting result, but it is limited to spirals of 10′ in diameter and upwards, without reference to their inclination with the line of sight, and the hundreds of smaller spirals photographed down to 2′ diameter are neglected altogether. The present paper deals with both these questions. The statistical material is mainly that employed by the writer in a recently published paper on "The Planes of the Spiral Nebulæ in Relation to the Line of Sight,"* which includes spirals 3′ in diameter and upwards, collected chiefly from *L.O.*, 13, the *Helwân Bulletins*, 9, 15, 21, and 22, and southern objects between $-40°$ and $-90°$ from the Franklin-Adams plates. The spirals down to 2′ diameter are here included, together with some additional objects of larger dimensions. The examples are plotted on polar graphs of each galactic hemisphere, as in *M.N.*, 81, Plate 4, but as the longitudes 50°, 140°, 230°, and 320° are evidently of fundamental importance, the graphs have been rotated so that 50° and 230° are horizontal, and the longitudes of the apex and antapex vertical. For the present purpose this form of graph has great advantages over the usual rectangular graphs, in which the relative positions are greatly and unnecessarily distorted.

As in the previous paper the spirals are divided into three classes, according to their major diameters:—

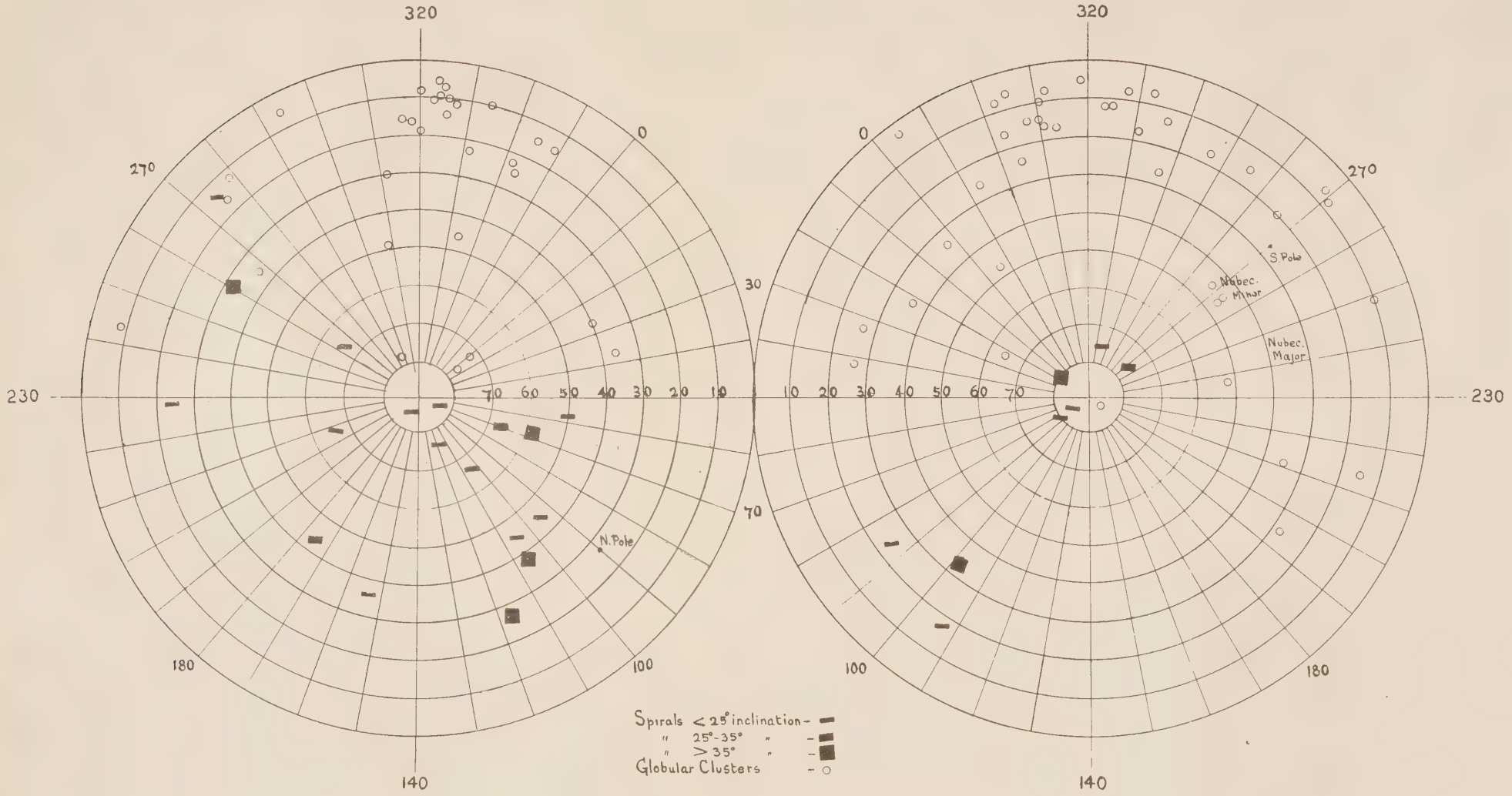
10′ and upwards

5′ and $<10'$

2′ and $<5'$

To distinguish the three classes, squares and rectangles of diminishing size are used. Squares represent spirals inclined at angles greater than 35° with the line of sight, two to one rectangles represent angles between 25° and 35°, and four to one rectangles indicate angles less than 25°.

* *M.N.*, 82, 510.



GALACTIC DISTRIBUTION OF THE LARGE SPIRAL NEBULAE AND GLOBULAR CLUSTERS.

Galactic Distribution of the Large Spirals and the Globular Clusters.

The first diagram gives the spirals of 10′ diameter and upwards, and the globular clusters for both hemispheres. The clusters are those derived from the Franklin-Adams plates, by Melotte (*Mem. R.A.S.*, **60**, 5); those marked "doubtful" by him being excluded, as it is quite possible some of these may be globular nebulæ and not clusters at all. It will be seen that the globular clusters and the spiral nebulæ fall for the most part into the two separate hemispheres, bounded by galactic longitudes 50° and 230°. The following points are to be noted:—

(1) The spirals of largest angular diameter in the North Galactic Hemisphere are found principally in the region of Ursa Major, about a median galactic longitude of 100°. *N.G.C.* 2403, 3031, 4258, and 5457 in this region are all 16″ in diameter. Many of the spirals 10′ diameter and upwards lie along 100°, and form part of a well-marked band of nebulæ passing over the north galactic pole, which comes out conspicuously if the spirals ranging down to 2′ diameter are plotted together.

(2) The number of spirals 10′ and over dealt with in the N.G.H. is 18, the planes being inclined to the line of sight as follows:—

		$n = N(\sin\theta_1 - \sin\theta_2)$.
< 10	10	3·1
$10° - 25°$	3	4·5
$25° - 35°$	1	2·7
$35° - 90°$	4	7·8

Six out of the ten under 10° are actually under 5°, as an examination of the individual spirals will show. Three of the four between 35° and 90° are in the Ursa Major region, viz.: *N.G.C.* 2403, 3031, and 4258. The fourth *N.G.C.* 5236 is far to the south of the galactic pole, and therefore out of reach in these latitudes.

(3) The largest spirals in the South Galactic Hemisphere lie between galactic longitudes 70° and 110°, extending beyond the pole about 20°. This corresponds to the direction of the largest spirals in the N.G.H., so their actual distribution in both hemispheres is roughly along the median longitude of 100°. It is thus evident that galactic longitude is of primary importance, latitude being in fact secondary. In each hemisphere the nebulæ approach quite close to the Galaxy about the longitude of 100°, the Andromeda Nebulæ (*N.G.C.* 224, S.G.H.) being in galactic latitude $-21°$, and *N.G.C.* 2403 in $+29°$.

(4) It has already been pointed out in a previous paper that the spirals of very large angular diameter are all concentrated in the S.G.H., although the number of spirals of 10′ and upwards is considerably less than in the N.G.H. The number of these in the S.G.H. is 8, divided as follows:—

		$n = N(\sin\theta_1 - \sin\theta_2)$.
$> 10°$	4	1·3
$10° - 25°$	1	2·0
$25° - 35°$	1	1·2
$35° - 90°$	2	3·4

(5) The number of globular clusters in the N.G.H. is 30, against 38 in the S.G.H. Their diameters range from 1′·5 to 45′,

$>10'$	17
$5'-10'$	27
$<5'$	24

In addition to these, 13 examples are given in *L.O.*, **13**, and the *Helwân Bulletins* which are not in Melotte's list. Six of these are in the N.G.H., and seven in the S.G.H. All of them fall in the hemisphere 230° to 50° (through 360°).

Galactic Distribution of the Spirals 5′ and <10′.

(6) The general distribution of spirals between angular diameters 5′ and <10′ in the N.G.H. is similar to that of the large spirals over 10°, but more extended. In particular, they are situated on each side of the median longitude 100°, and beyond the pole along 280°. This band is a well-marked feature in this hemisphere and comes out prominently on the second diagram. With the exception of this band, there are only five examples of these dimensions in the two quadrants 230° to 50° (through 360°). Another characteristic of the N.G.H. is the great scarcity of spirals in the quadrant 320° to 50° (through 360°), although the whole of the quadrant has been thoroughly explored and contains well-known northern constellations.

(7) The number of spirals of these dimensions in the N.G.H. is 48, of the following inclinations to the line of sight:—

		$n=N(\sin\theta_1-\sin\theta_2)$.
$<10°$	16	8·3
$10°-<25°$	11	11·9
$25°-35°$	6	7·2
$35°-90°$	15	20·5

(8) The number of these dimensions in the S.G.H. is 35, of the following inclinations to the line of sight:—

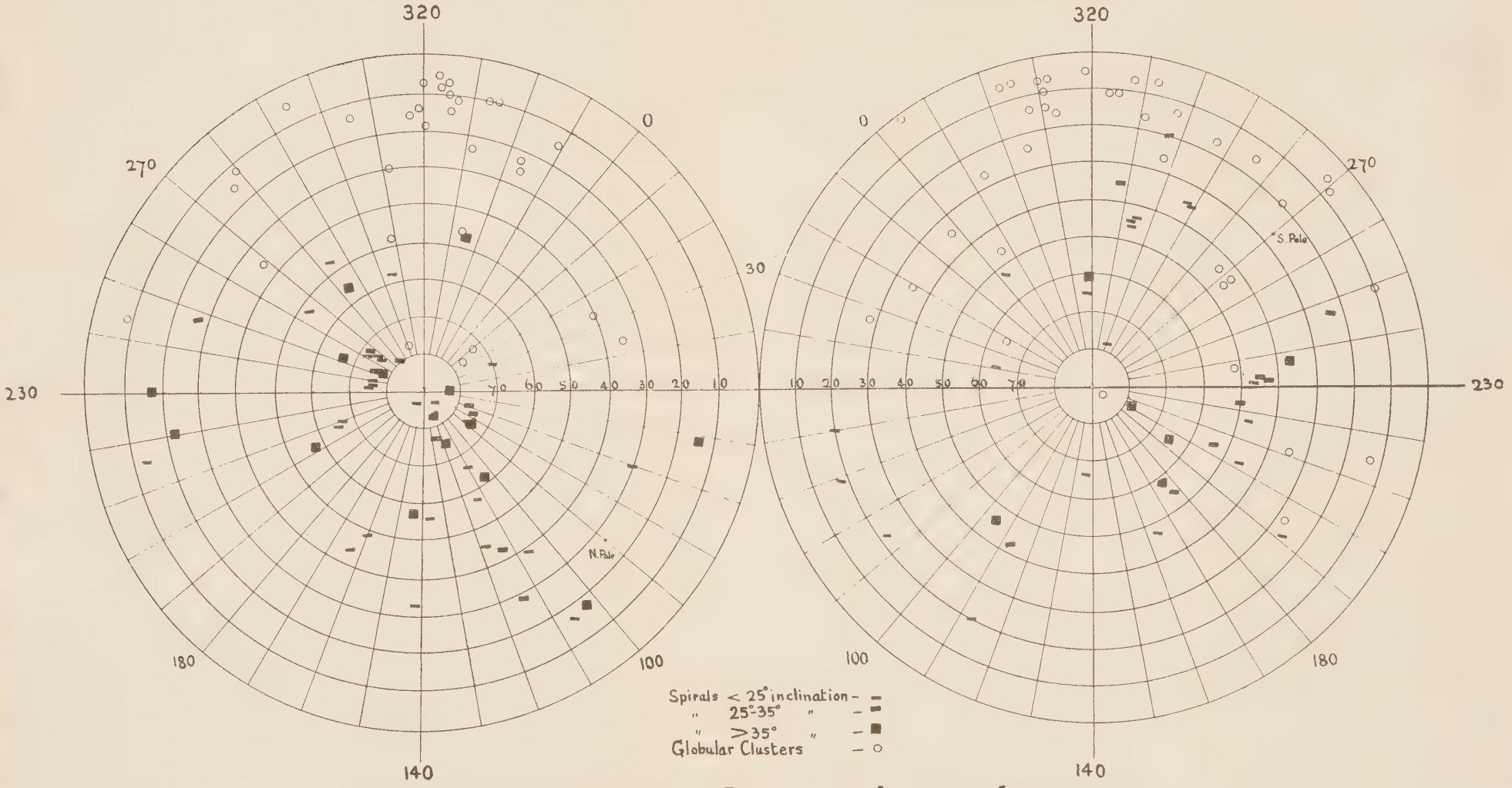
		$n=N(\sin\theta_1-\sin\theta_2)$.
$<10°$	16	6·0
$10°-25°$	4	8·7
$25°-35°$	9	5·3
$35°-90°$	6	15·0

The proportion of angles less than 25° is thus considerably more than in the N.G.H.

(9) Although the south galactic pole region is conspicuous for a concentration of 5 large spirals, there are very few smaller spirals, and most of those $5'<10'$ in diameter lie in two regions away from the pole, one stretching from the Andromeda region towards $\lambda 250°$, $\beta-0°$, and the other lying between the pole and the galactic equator in longitudes

NORTH GALACTIC HEMISPHERE.

SOUTH GALACTIC HEMISPHERE.



GALACTIC DISTRIBUTION OF SPIRALS 5′ TO <10′ DIAMETER.

290° to 320°. The latter occurs in an area free from globular clusters and is well defined in its limitations. It is the only exception to the general concentration of the spirals towards and in the other hemisphere, the few remaining spirals in the hemisphere of which the apex is λ320 being sporadic. There is a corresponding scarcity of spirals as in the N.G.H. in the quadrant 50° to 320° (through 360°).

(10) An examination of the F.A. plates containing spirals in the northern hemisphere of the sky, shows conclusively that most of the larger spirals over 5′ diameter have already been photographed on a larger scale with reflectors; only one example of these dimensions not already known, or at any rate published, was found. The principal area which has so far not yet been explored by such reflectors is in the Zone −40° to −90°. The F.A. plates of this area have been examined by the writer, and the result already published.

(11) There seems to be no general galactic distribution of spirals inclined at a small angle with the line of sight, as these appear in all regions containing spirals. At the same time, there can be no doubt that they are grouped together in small areas, both as regards dimensions and inclinations.

Galactic Distribution of Spirals 2′ to 5′ (Third Diagram).

(12) The number of spirals of these dimensions in both hemispheres is considerable, and probably only about 70 per cent. of the total have been dealt with in the present paper. In the N.G.H. they are fairly widespread, and extend well into the semi-hemisphere 50° to 230° (through 360°). At the same time there are two principal concentrations, each of about 10 square degrees, one about a centre at λ110°, β + 70°, and the other at λ280°, β + 70°; these concentrations lying on each side of the north galactic pole. There are, in fact, so many in these areas that it was not possible to insert them all in the diagram.

(13) The number recorded in this hemisphere is 165, divided as follows:—

		$n = N(\sin\theta_1 - \sin\theta_2)$.
<25°	77	69·6
25°−35°	33	24·9
>35°	55	70·4

(14) There is a remarkable absence of the small spirals in the region +50°, and the galactic equator along longitudes 90° to 230°.

(15) In the S.G.H. 111 spirals of these dimensions have been included.* As in the N.G.H. they are fairly widespread, and also extend well into the semi-hemisphere bounded by λ50° to 230° (through 360°). They are inclined to the line of sight in the following proportions:—

		$n = N(\sin\theta_1 - \sin\theta_2)$.
<25°	42	46·8
25°−35°	37	16·7
>35°	32	47·3

* A further 16 examples have been included in the diagram.

They lie for the most part in the regions containing spirals of 5′ to < 10′ diameter.

General Conclusions.

There seems to be no doubt that, broadly speaking, spirals and globular clusters are mutually exclusive. The hemisphere of which the pole is λ320°, β0° contains 90 per cent. of the globular clusters, and only rarely are clusters and spirals found in the same regions. In the N.G.H. the globular clusters are all concentrated in the semi-hemisphere 50° to 230° (through 360°), and there are very few sporadic spirals in this area. In the S.G.H. there is more intermingling of the spirals and clusters. As the angular diameter of the spirals decreases their inclination to the line of sight, small in the case of the larger spirals, tends to approach more nearly to the average of 30°. There is also a tendency for the spirals of inclinations greater than 30° to appear in regions away from the poles.

The diameters and magnitudes of globular clusters have been correlated with distance by Shapley, and the results are in good agreement. There seems to be no valid reason why the same criterion should not be applied to the spirals. As undoubted spirals have already been photographed down to 30″ diameter, and the Andromeda Nebulæ is 130′, we must conclude that these formations stretch to a vast distance from the galactic plane.

The spirals in the N.G.H. are much more numerous than in the S.G.H., and there is very definite evidence of a band, fairly widespread in the Ursa Major region, stretching past the pole beyond Virgo, the average size becoming smaller as the band passes beyond the pole. In the S.G.H. the number of spirals is only two-thirds of the number in the N.G.H.; they are much more widely spread, and contain the five largest spirals in the whole sky.

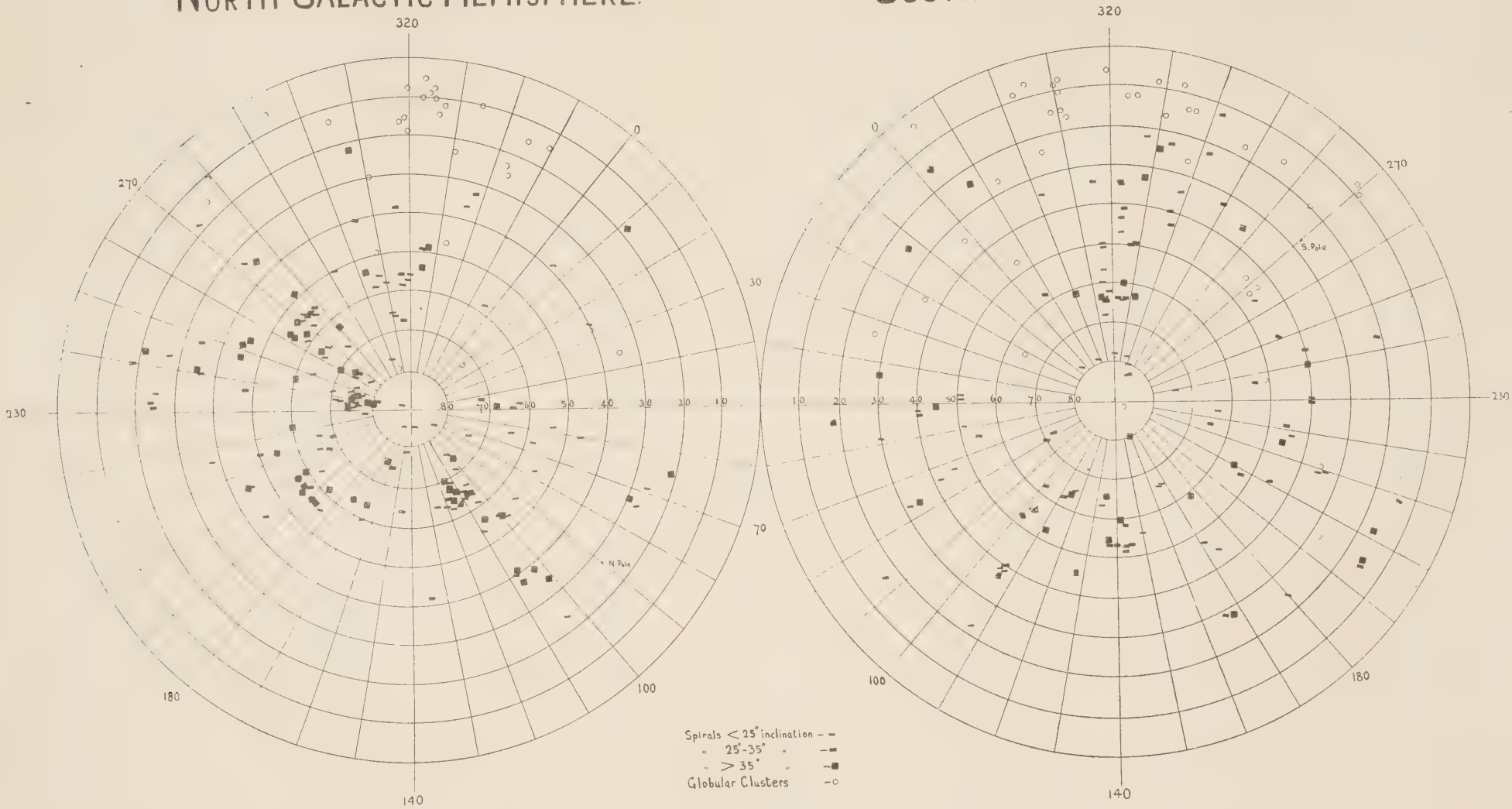
These facts suggest that the galactic plane is situated excentrically with regard to the spirals, those in the S.G.H. approaching the plane more closely than those in the N.G.H.

The quarter of the sphere bounded by λ320° to 50° (through 360°) in both the north and south galactic hemispheres is conspicuously free from spirals right up to the poles. No reason for this feature of distribution can be readily suggested, but the same region contains 50 globular clusters out of a total of 81.

We are still without sufficient information about the small spirals and globular nebulæ to make any good estimate of their distribution; the examples already photographed, however, are situated in the same regions as the spirals of larger dimensions.

NORTH GALACTIC HEMISPHERE.

SOUTH GALACTIC HEMISPHERE.



GALACTIC DISTRIBUTION OF SPIRALS 2′ TO < 5′ DIAMETER.

Note on the Corona of 1908. By J. Evershed, F.R.S.

In the *Memoirs of the Kodaikánal Observatory*, vol. i. part ii., p. 67, the corona of 1908 is referred to as affording a test of the relationship of prominences to coronal streamers, since our Kodaikánal prominence photographs show considerable activity over the sun's south polar region and an almost complete absence of prominences over the north pole. This refers to the average conditions over each of the years 1907 and 1908. The eclipse occurred on 1908 January 3, but our photographs taken early in January, and one taken seven hours after the eclipse itself, do not show any temporary departure from this distribution.

The corona, as drawn by the late Mr. Wesley from photographs obtained at Flint Island by Mr. McClean,* shows the opposite distribution as regards the coronal streamers, which are more conspicuous over the north polar region than over the south, and thus the test in this instance appears to fail, although from previous eclipses there seemed to be a close correspondence between the coronal streamers and the principal zones of prominence activity. In view of this we were constrained to doubt whether the Flint Island plates were correctly oriented, and recently I have discovered evidence that shows, I think conclusively, that the orientation is incorrect, the north and south points being interchanged.

In the *Publications of the Astronomical Society of the Pacific*, **20**, 71, Dr. Campbell publishes a photograph of the corona, taken at the same time and place as Mr. McClean's photographs, and in this a bright prominence 2′ high is shown in the south-west quadrant at position angle about 212°, while Mr. Wesley refers to what is obviously the same prominence in the north-west quadrant. Again, Dr. Campbell refers to a coronal structure on the east limb radiating from a point within the disc at P.A. 75°, but this structure is plainly seen in Mr. Wesley's drawing at about P.A. 96°. These differences can be brought into harmony only by interchanging the north and south limbs.

The positions of the north and south points may be tested in another way. Mr. Wesley states that his orientation was determined from the point of reappearance of the sun at third contact, showing clearly on plate No. 9 of the series. But this of course only defines two opposite points on the eclipsed sun, one west and the other east, north and south remaining ambiguous. Now at all eclipses the point of third contact is in a clockwise direction from the sun's north point (or the north pole) as seen in the sky from any part of the earth ; and this holds also in prints taken from negatives photographed with the camera pointing direct at the eclipse. But at Flint Island Mr. McClean used a mirror, and this reverses the direction, so that the north point must be in a clockwise direction from third contact. If, therefore, we take the reproductions in Mr. McClean's report to be unreversed copies similar to ordinary prints, as is highly probable, then, as the point of third contact is shown on the right-hand side in plate 10, obviously the north point should be at the

* *Report of the Solar Eclipse Expedition to Flint Island*, 1908 Jan. 3, by F. K. McClean, F.R.A.S., and others.

bottom and not at the top, as shown. The same applies to all the other photographs in the *Report*, and to Mr. Wesley's drawing.

If the north and south limbs are interchanged in the drawing on a diameter passing through the point of third contact there will be a practical agreement with Dr. Campbell's orientation, the prominence will then be in the south-west quadrant, and the point of convergence in the coronal structure on the east limb will be at about P.A. 70°. Also the unsymmetrical position of the solar axis to which Mr. Wesley calls attention as an anomalous feature will be greatly reduced.

Finally, I have compared the eclipse photographs with a prominence plate taken here on January 4, $8^h\ 53^m$ I.S.T., or about seven hours after the eclipse plates were exposed. Owing to changes during this interval in the form of the bright eruptive prominence recorded at the eclipse, the evidence is not very decisive, but strongly favours Dr. Campbell's orientation, for a high prominence of the eruptive type is seen on the south-west limb, P.A. 198° to 210°, whilst all other prominences are small.

I conclude therefore that Dr. Campbell's orientation is correct and that the corona of 1908 is not an exceptional case, but, on the contrary, adds weight to the evidence previously found for a close relationship between prominences and coronal streamers, as advocated by Major Lockyer in his recent and also his earlier paper.*

In Mr. Wesley's beautiful series of coronal drawings this error of orientation of the 1908 eclipse should be noted and corrected, as otherwise it may prove very misleading in any future researches.

The Observatory, Kodaikánal:
1922 *December* 12.

Micrometrical Measures of Double Stars (17th Series).

By Rev. T. E. Espin.

In the Measures of known Doubles, and in the list of New Doubles found with the 24-inch, the letter M indicates that the measures were made by Mr. Milburn with the 17¼-inch. In Mr. Milburn's list the stars marked E were measured with the 24-inch by me.

Name.	R.A. (1900).	Decl.	P.	D.	Mags.	Nts.	Date.
	h m	° ′	°	″			1922
Gr. 34	0 12·7	+43 27	225·7	29·24	8·0 11·0	1	·928
Espin 226	49·9	38 27	275·1	6·14	9·5 13·0	2	·995
Espin 154	53·9	37 14	70·7	6·33	8·5 9·2	3	·992
Σ 152 rej.	1 39·2	60 44	103·4	9·35	9·0 10·5	2	·119
B.D.+37°·385	44·8	37 28	213·1	4·26	8·8 13·0	2	·896

* *Monthly Notices of the Royal Astronomical Society*, **63**, 481 ; **82**, 323.

Name.	R.A. (1900).	Decl.	P.	D.	Mags.	Nts.	Date.	
	h m	° ′	°	″			1922	
Espin 159	1 45·9	+37 17	20·6	9·00	8·9 13·7	4	·950	
Σ 485	3 59·0	62 4	303·7	18·05	6·0 6·1	3	·542	M.
Σ 503	4 7·9	63 53	226·0	4·37	9·3 9·3	2	·147	
Σ 538	21·3	64 0	218·7	7·14	8·6 9·0	3	·190	AB.
			358·0	10·90	C=13·0	3	·190	AC
Σ 633	5 1·3	63 29	341·5	12·04	7·0 9·5	2	·187	
Σ 656	11·2	63 5	215·4	2·62	8·5 9·2	2	·187	
Σ 677	15·3	63 18	236·2	1·35	8·3 8·7	2	·206	
Σ 866	6 8·9	62 14	196·7	18·22	8·5 9·1	2	·191	
Σ 882	14·1	64 59	267·9	3·58	8·5 10·0	4	·181	
Σ 887 rej.	15·3	60 10	175·9	9·28	9·6 10·5	2	·178	
Σ 1006	57·6	62 42	73·0	29·65	7·0 8·0	2	·187	
Σ 1122	7 36·4	65 24	5·0	15·32		1	·186	M
Σ 1125	37·0	61 8	341·2	22·77	8·6 9·1	3	·185	
Σ 1127	37·7	64 19	340·0	5·04		1	·194	M, AB
			174·3	11·00		1	·194	M, AC
Σ 1350	9 26·0	67 14	247·6	10·08		1	·189	M
Holmes	17 50·8	39 30	278·6	4·91	8·4 11·0	3	·666	
h 1335	18 35·7	35 13	8·3	15·09	10·1 11·2	2	·634	
h 1360	57·2	36 21	226·6	9·80	11·0 11·2	3	·638	
Holmes	19 7·9	38 40	205·5	6·74	10·5 10·8	2	·686	
Scheiner 588	14·7	38 59	80·3	2·10	11·1 12·5	4	·739	
Scheiner 602	23·1	39 22	160·2	3·58	9·2 9·4	3	·812	
h 2951	20 14·9	39 37	123·9	10·79	8·8 9·3	2	·728	
Σ 2731	45·3	39 25	85·4	4·18	8·5 9·2	2	·859	AB
			173·6	16·90	C=12·8	4	·861	AC
			245·5	18·67	D=12·5	4	·861	AD
Espin 207	21 7·2	37 56	250·3	2·10	9·5 9·6	2	·957	
Σ 2788 rej.	14·2	66 56	354·7	7·62	8·5 9·7	2	·728	M
Scheiner 1506	16·0	39 32	81·9	4·94	10·2 10·9	4	·827	
Espin 259	28·2	38 22	316·4	3·31	9·5 11·1	2	·957	
Espin 260	28·5	38 35	277·6	5·39	9·2 13·3	4	·919	
Espin 211	46·5	39 17	192·4	2·47	9·5 10·5	3	·966	
h 1759	22 18·9	38 41	300·5	11·32	10·8 11·2	3	·638	
Espin 392	31·4	39 6	226·2	9·66	8·5 12·5	1	·983	Dd 8 Lacertae
Espin 216	33·2	36 10	1·4	2·24	9·5 10·7	2	·957	BC
Espin 217	40·9	36 23	61·9	3·91	9·3 9·7	3	·980	BC
h 1838	54·7	66 36	269·1	2·12	8·0 8·5	2	·943	M
OΣΣ 243	23 5·3	36 18	330·6	11·82	C=13·5	2	·972	BC

Notes.

Gr. 34. This is the *comes* measured in 1917, which is thus proved not to belong to the system.

Σ 152 rej. The three stars Σ 151, Σ 152, Σ 153 are all described in the "Catalogus Novus" as Class II., and are bracketed together with the note :—

"Tres duplices in acervo fere in linea. Secundae aequalis a linea longius distat. Ad tertiam est major vicina praecedens" (p. 4). The three stars are :—

		h m s	°	′		
Σ 151 rej.	B.D. +60°·332	1 36 5·7	+60	28·6	9·5 (1855), with B.D. 60°·334	
Σ 152 rej.	B.D. +60°·333	11·1		29·9	9·0	
Σ 153 rej.	B.D. +60°·343	38·9		31·7	9·1	

Σ 151 rej. is evidently identical with Dunér No. 1 (β's. G.C. 909), and β has wrongly identified Σ 152 rej. with B.D. +60°·336 (β's. G.C. 908), his measures, with distance 32″·57, showing that this star could not be called Class II.

B.D. +37°·385. This star was measured by Aitken for Espin 159.

Σ 503. Identified by Lewis as B.D. +63°·489, but it is *sp* this star and not in the B.D.

Σ 1006. A *comes*, mag. 14·0; P. 306°·5; D. 37″·84. Single setting.

Holmes $17^h\ 50^m·8$. This is B.D. +39°·3259.

Holmes $19^h\ 7^m·9$. This is not in the B.D. It lies $6^s·3$ *f* Σ 2481, 4′*n*. This and the previous star are taken from a MS. Catalogue I have made of the late Mr. Holmes' double stars, collected from various letters of his in the *English Mechanic*.

Scheiner 588. P. 73°·4; D. 1″·78; mags. 11·0, 11·0 1896·5.

Scheiner 602. Measured later as J. 1157; perhaps increase in angle.

h 2951. Angle perhaps decreasing.

Scheiner 1506. When measured in 1917 I called attention to the difference between the measures and Scheiner's. The measures this year show a further decrease of angle, and increase of distance.

Espin 211. The 24-in. shows *comes*, 14·0 mag.; P. 170°·3; D. 8″·95; and another 13·2 mag.; P. 96°·6; D. 20″·58.

New Double Stars found with the 24-inch Reflector.

No.	B.D.	R.A. (1900).	Decl.	P.	D.	Mags.	Nts.	Date.	
	°	h m	° ′	°	″			1922	
1934		0 1·9	+62 36	68·9	4·75	9·2 10·8	2	·004	
				69·4	4·55	9·5 11·5	2	·875	M
1935	+36·5155	2·1	37 7	134·0	8·64	8·9 10·4	2	·873	
1936	+38·21	11·5	38 37	75·5	6·02	9·3 10·5	2	·862	
1937	+60·34	15·9	61 2	308·9	4·60	9·5 14·0	2	·004	
1938	+36·52	20·0	36 51	9·3	7·24	9·4 9·8	3	·965	
1939	+38·54	23·4	38 33	274·3	6·06	8·3 12·0	2	·836	
1940	+37·86	28·1	37 16	340·6	4·23	9·5 11·5	3	·901	
1941	+37·92	29·3	37 44	346·0	2·60	9·2 9·3	2	·865	
1942		38·1	38 57	276·9	2·92	9·4 11·1	2	·931	
1943	+37·154	46·2	37 49	135·2	4·95	9·5 10·5	2	·862	
1944		47·5	38 0	106·5	3·27	10·7 11·5	2	·868	
1945	+60·168	1 2·5	61 13	163·7	3·58	9·3 9·5	4	·017	

No.	B.D.	R.A. (1900).	Decl.	P.	D.	Mags.	Nts.	Date.	
	°	h m	° ′	°	″			1922	
1946	+38·196	1 3·1	+38 58	134·0	4·26	9·2 11·2	3	·855	
1947	+37·245	10·5	37 35	43·7	6·54	9·0 14·0	3	·956	
1948	+37·288	22·7	37 50	242·4	4·76	9·5 10·5	2	·931	
1949	+60·291	31·9	60 59	65·1	5·95	9·5 12·5	2	·101	
1950	+60·339	39·5	60 45	77·0	1·92	12·0 12·5	2	·125	BC
				75·7	2·20		4	·921	BC–M
				253·3	28·36	A= 8·6	2	·125	AB
				252·9	28·78		4	·896	AB–M
1951	+60·354	42·4	61 5	159·0	6·00	9·3 9·4	3	·127	
1952	+60·433	59·7	60 28	223·6	5·80	9·3 11·0	3	·123	
1953	+37·487	2 2·8	38 13	275·7	2·63	10·0 10·1	4	·908	
1954		18·7	60 39	124·0	3·93	10·1 11·5	2	·130	BC
				125·3	3·95		2	·960	BC–M
				52·6	22·63	A= 9·9	2	·130	AB
				53·4	21·62		2	·969	AB–M
1955	+37·558	22·2	38 3	20·2	8·76	8·5 11·5	3	·932	AB
				335·3	9·16	C–13·0	3	·932	AC
				276·9	7·12		3	·956	BC
1956	+38·531	33·6	38 24	206·8	1·94	9·5 11·0	3	·960	AB
				72·3	3·79	C=12·0	4	·918	AC
1957	+61·571	3 16·9	61 44	171·3	4·47	9·5 12·0	3	·111	
1958	+60·727	32·7	60 44	187·9	3·37	8·7 11·6	2	·123	
1959	+61·657	53·0	61 9	24·8	2·85	9·4 11·6	3	·111	
1960	+61·705	4 14·1	61 26	103·2	2·67	9·5 10·1	2	·101	
	+64·439	15·2	64 9	266·7	11·99	9·0 9·2	2	·175	
1961	+64·441	17·6	64 8	135·5	3·97	9·5 10·6	2	·175	
1962	+64·455	22·1	64 6	60·8	7·01	9·0 11·0	2	·175	
1963	+61·770	5 5·2	61 20	271·9	3·77	8·8 12·5	2	·138	
1964	+64·519	10·8	64 30	259·9	5·20	10·2 10·3	2	·183	
1965	+36·3376	18 58·6	36 31	242·1	7·28	9·2 11·0	2	·634	
1966	+39·3696	19 11·3	39 8	331·6	4·58	9·2 12·8	3	·726	
1967	+39·3840	33·1	39 46	234·6	5·56	10·0 11·0	3	·664	BC
				60·9	23·92	A= 9·2	2	·654	AB
				308·6	10·51	D=14·0	3	·684	AD
1968	+39·3858	35·4	39 56	277·6	4·16	10·5 11·7	3	·650	BC
				115·5	41·19	A= 8·8	2	·652	AB
1969	+38·3727	39·9	38 41	170·6	2·48	9·1 9·4	3	·831	
1970	+39·3986	55·6	39 26	328·0	4·74	10·7 12·1	3	·746	BC
				148·7	19·11	A= 9·3	3	·746	AB

No.	B.D.	R.A. (1900).		Decl.		P.	D.	Mags.		Nts.	Date.	
	°	h	m	°	′	°	″				1922	
1971	+39·4037	20	3·3	+39	21	287·1	2·81	9·2	9·4	2	·809	
1972	+39·4048		4·5	39	9	109·6	5·18	9·4	10·5	2	·865	
1973			8·2	39	20	358·3	4·27	10·0	12·2	4	·846	
1974			12·2	39	19	175·0	3·72	9·3	12·0	3	·845	
1975	+38·4084		22·6	39	5	34·8	4·20	9·5	11·0	2	·845	
1976			23·6	39	29	79·2	3·51	10·2	10·5	2	·715	
1977			29·0	39	35	28·2	2·54	10·6	11·5	3	·693	
1978	+39·4258		34·2	39	44	134·5	3·24	9·5	10·5	2	·654	
1979	+39·4264		35·0	40	3	301·5	7·11	9·1	13·0	2	·634	
1980	+39·4267		35·5	39	56	62·0	7·56	9·4	12·0	2	·643	
1981	+38·4210		40·4	39	5	295·6	3·00	9·3	11·3	2	·845	
1982	+39·4312		42·9	39	58	297·0	4·43	9·2	9·5	2	·684	
1983	+38·4276		51·2	38	51	268·2	5·41	8·5	14·0	2	·862	
1984			51·6	38	47	114·3	3·32	10·5	11·0	3	·882	
1985	+38·4300		54·2	38	55	293·0	6·27	9·6	12·0	4	·925	
1986	+39·4420		59·3	40	0	32·4	4·26	8·7	12·0	2	·634	
1987	+38·4338	21	1·2	38	50	355·2	6·58	9·1	10·7	2	·929	
1988	+38·4340		1·9	39	5	271·8	4·95	9·5	12·0	2	·913	BC
						314·9	27·88	A=	9·0	2	·913	AB
1989	+39·4439		3·0	39	57	169·1	4·66	9·3	9·6	2	·643	
1990	+39·4442		3·4	39	34	40·7	5·91	9·5	12·0	3	·836	
1991	+38·4377		7·3	38	30	25·4	2·62	9·5	10·0	1	·961	
1992	+38·4383		7·9	39	6	204·5	3·26	9·5	12·0	2	·890	
1993	+38·4438		14·2	38	25	272·8	6·93	9·4	14·0	3	·931	
1994	+39·4544		20·2	39	36	104·2	4·97	9·5	11·5	2	·715	
	+38·4545		31·1	38	12	97·1	17·26	8·6	9·1	2	·932	
1995			31·3	38	10	50·3	2·12	10·6	11·2	2	·932	
1996			46·0	37	22	167·2	2·17	10·6	11·1	4	·927	
1997		22	39·1	39	40	87·4	4·28	10·2	10·5	2	·834	
1998	+37·4731		52·6	38	0	92·4	5·00	9·3	12·2	3	·855	
1999			59·5	37	20	64·8	2·97	10·0	10·2	3	·925	
2000	+39·5003	23	1·0	39	33	67·1	7·91	8·2	12·2	2	·736	
2001	+36·3050		16·2	36	36	295·3	6·30	9·3	10·6	2	·929	
2002	+37·4832		19·4	37	20	98·4	4·54	9·5	10·0	2	·862	
2003			26·4	36	26	264·4	4·79	9·5	10·7	3	·931	
2004			49·1	37	4	350·1	4·14	10·0	10·1	3	·885	
2005	+36·5129		49·3	36	27	155·1	5·78	9·3	10·5	2	·957	
2006			59·3	62	30	267·5	2·01	10·0	11·1	4	·017	

Notes.

1944. In field with B.D.+37°·158 at P. 115°.
1950. *Comes*, 13 mag. S., a more distant one *np*.
1956. Thought to be nebulous on one night.
1968. This is A of Scheiner 672.
1970. AB is Scheiner 768, P. 148°·5, D. 19″·32; 1896·5.
1977. 51″ from B.D.+39°·4228 at P. 5°.
1985. This is B of Scheiner 1338, P. 93°·6, D. 26″·28; 1895·6, now P. 98°·0, D. 31″·99, 2 nts. 1922·925. B is apparently the B.D. star, and if the measures are correct A must have a P.M. of 0″·2 towards P. 296°.
1988. A is Aitken 2691.
2003. In field with B.D.+36°·5075 at P. 345° D. 67″.

The following Double stars have been found with the 17¼-inch reflector by Mr. Milburn, and measured by him, except those marked E which were measured with the 24-inch.

No.	B.D.	R.A. (1900).	Decl.	P.	D.	Mags.	Nts.	Date.
	°	h m	° ′	°	″			1922
242	+64·1896	0 0·3	+64 40	72·7	3·72	9·5 9·7	2	·838
243	+64·26	13·6	64 40	308·6	4·81	9·2 10·5	2	·932
244		15·3	63 52	283·6	1·91	10·0 14·0	2	·997
245		23·8	65 0	170·1	3·39	9·5 11·5	3	·882
246		57·3	63 9	220·6	3·38	11·0 12·0	3	·985
247		59·2	62 21	297·8	3·49	10·0 10·5	2	·997
248		1 0·8	62 53	325·3	3·77	10·6 11·6	2	·011 E
249		3·7	62 20	197·9	3·38	10·0 11·0	4	·908
		4·2	64 43	20·4	11·03	9·0 10·5	2	·838
250		42·2	61 23	294·7	4·40	11·0 11·2	4	·955
251		47·0	61 53	58·2	2·63	10·0 10·1	3	·891
252	+62·337	54·5	62 44	298·6	2·55	9·5 14·0	5	·998
253		58·1	62 46	41·8	3·61	10·0 10·5	2	·935
254	+64·364	2 56·5	64 15	192·4	6·60	9·5 11·0	2	·943
255		3 8·9	63 37	58·4	6·80	9·5 11·5	3	·948
256	+63·463	47·8	63 26	273·1	4·81	9·2 9·3	2	·101 E
257	+65·429	4 38·4	65 10	325·2	5·73	9·5 11·5	1	·909
258	+64·550	5 43·3	64 57	277·5	4·22	9·0 12·0	3	·155 E
259		59·4	64 25	240·5	4·25	10·9 11·2	3	·161 E
260	+65·540	6 31·9	65 21	65·0	5·56	8·5 11·2	3	·161 E
261	+65·573	7 20·9	65 44	77·5	3·41	9·0 11·5	2	·183 E
262	+60·1093	46·6	60 1	259·0	7·05	9·0 12·5	2	·183 E
263		46·8	60 37	265·4	3·01	10·0 10·2	2	·183 E
264		8 24·3	65 20	233·1	2·27	10·4 10·5	2	·188 E
265	+66·1247	19 48·5	66 31	188·3	3·03	9·5 10·0	4	·693
266	+66·1298	20 26·5	66 46	180·4	3·28	9·5 12·8	4	·689

No.	B.D.	B.A. (1900).	Decl.	P.	D.	Mags.	Nts.	Date.
	°	h m	° ′	°	″			1922
267		20 47·7	+65 25	225·1	2·28	10·0 11·0	4	·793
268	+66·1343	55·9	66 22	336·1	4·84	8·0 10·5	2	·845
269	+65·1641	21 42·1	65 53	358·2	2·27	9·0 9·2	3	·730
270	+65·1653	45·7	65 57	257·4	1·86	9·5 10·0	3	·812
271		22 1·1	65 22	147·9	3·02	10·0 14·0	2	·942
272		29·6	67 6	270·7	2·86	11·0 11·2	2	·931
	66·1523	29·9	66 40	152·9	11·49	9·0 11·0	2	·932
273	+66·1533	36·7	66 19	202·4	7·61	9·5 10·5	3	·669
274		44·8	65 28	357·4	4·32	10·0 10·5	4	·728
275	+65·1828	50·0	65 34	281·1	5·85	9·5 11·5	3	·918
	+65·1875	23 13·9	65 36	200·3	9·52	9·5 10·0	2	·913

Notes.

246. At P. 310°·5, D. 31″·24 from B.D.+62°·191.
256. A faint *comes* at P. 20°±.

Stars Missing or Faint.

	°	h m s	° ′			
B.D.+36·3		0 1 19·1	+36 30·3	9·5	1855.	1922	Dec. 2, not seen in moonlight; Dec. 9, Dec. 15, about 12 mag.
B.D.+38·55		0 21 6·2	+38 2·4	9·5	1855.	1922	Nov. 1, not seen in moonlight; Nov. 15, Dec. 18, nothing here above mag. 14.
B.D.+38·84		0 32 11·8	+38 3·8	9·5	1855.	1922	Nov. 1, not seen; Dec. 6, 18, nothing above 13 mag. here.
B.D.+37·244		1 8 5·6	+37 28·8	9·5	1855.	1922	Dec. 8, Dec. 15, no star seen.

Observations of Novæ, 1922. By W. H. Steavenson.

In this paper are given photometric observations of certain novæ made during 1922. The results for the previous year, relating to the same stars, were published in *M.N.*, 82, 49. Except in the case of Nova Persei (1901) nearly all the observations were made with the 6-inch Wray refractor. Some of the novæ (notably Nova Cygni 1876) are really too faint for satisfactory work with this aperture, and observations by those possessing larger instruments would seem very desirable, in view of the general neglect of novæ in their later stages.

Nova Ophiuchi (1848).

Observed on 27 nights between 1922 May 15 and August 27.

Date.	Mag.	Date.	Mag.	Date.	Mag.
1922 May 15	12·4	1922 June 11	12·5	1922 Aug. 7	12·6
21	11·95	15	12·55	8	12·7
22	12·1	20	12·5	16	12·6
23	12·05	29	12·6	17	12·6
24	12·05	July 19	12·15	20	12·45
26	12·35	20	12·1	23	12·1
27	12·3	21	12·1	24	12·0
28	12·4	30	12·6	26	12·1
29	12·3	31	12·5	27	11·95

These figures, so far as they go, are consistent with a period of about 50 days, with maxima on or about May 21 (July 10), and August 29. A similar period was suggested by the 1921 observations, and it is worthy of note that maxima observed on 1921 June 5 and July 25 exactly fit in with the results for 1922, giving respectively intervals of 350^d and 300^d to the maximum of 1922 May 21, corresponding to 7 and 6 periods of 50^d. Professor Turner, applying harmonic analysis to Barnard's observations of this star, found evidence of a period of $27^d{\cdot}3$ (*M.N.*, 81, 426). This period would satisfy very closely the maxima observed in 1921 June and 1922 May, giving 13 periods, but it can scarcely be reconciled with the intervening observations, or with the later ones of 1922. The same, of course, applies to a period of twice this length.

Moonlight and haze have greatly interfered with work on this nova. From the scanty data available it appears that the rise to maximum is sensibly more abrupt than the fall; but much more frequent observations are desirable in order to determine the exact shape of the light-curve.

The comparison stars used are the same as those employed by Barnard, who gives particulars of their positions and magnitudes in *M.N.*, 81, 167.

Nova Aquilæ (1918).

Observed on 81 nights between 1922 May 13 and December 26. This nova, whose magnitude was still falling slowly in 1921, has now apparently ceased to fade. The mean magnitude has been 9·98 throughout the period covered by the observations, which lie within $0^m{\cdot}1$ of this value. The small fluctuations are probably attributable to errors of observation, which are apt to be large in the case of certain novæ owing to the nebulous character of their images. With the low power always used for Nova Aquilæ (65) the image appears perfectly stellar, but the star is inclined to be unduly bright to averted vision.

If the nova is really variable the changes must be small. There is some reason to suspect the existence of a period of about 20 days, with an average range of less than $0^m.2$. In view of the uncertainty attaching to these results, it has not been thought worth while to publish the individual estimates. The chief point of interest is the stabilisation of the mean magnitude after an interval of less than 4 years from the date of outburst.

The comparison stars used are those contained in a Harvard circular published shortly after the first appearance of the nova as a bright object. It will be remembered that previous to this its photographic magnitude varied between 10 and 11.

Nova Cygni (1920).

Observed on 92 nights between 1922 May 4 and December 26. This nova, whose magnitude was less than 15 before the outburst, is still slowly fading. The following list of magnitudes represents the mean observed brightness at intervals of 10 days, each value given depending on the observations made during the preceding and following 10 days, making 21 days in all.

Date.	Mean Mag.	No. of Nights.	Date.	Mean Mag.	No. of Nights.
1922 May 10	10·34	7	1922 Sept. 7	10·89	8
20	10·43	15	17	10·91	7
30	10·53	14	27	10·99	4
June 9	10·64	9	Oct. 7	11·05	7
19	10·64	9	17	11·07	9
29	10·71	8	27	11·05	10
July 9	10·81	11	Nov. 6	11·07	8
19	10·85	12	16	11·11	4
29	10·88	12	26	11·17	2
Aug. 8	10·89	8	Dec. 6	11·20	1
18	10·88	8	16	11·13	6
28	10·87	7			

These figures, when plotted, show that the fall in brightness has not been perfectly steady, but that it seems to undergo a check at intervals of about 60 days. Such haltings in the fall occur at or near the following dates—1922, June 9, August 8, October 7, and December 6. Apart from the general downward tendency the variability of this nova is very slight, amounting to $0^m.05$ above or below a mean curve deprived of all small fluctuations. The nebulous character of the image of the nova makes it very difficult to estimate its brightness accurately. This point was dealt with in a paper communicated to the British Astronomical Association in 1922 June (*Journal of B.A.A.*, vol. xxxii. No. 8).

Nova Cygni (1876).

Observed on 7 nights between 1922 May 15 and September 17. There have been very few opportunities of observing this nova, which is only visible in the 6-inch telescope on the darkest and most transparent nights.

Date.	Mag.	Date.	Mag.
1922 May 15	14·81	1922 July 19	14·57
21	14·71	20	14·62
27	14·51	21	14·72
		Sept. 17	14·81

From these figures it seems probable that a maximum occurred some time during the month of June. Barnard observed a maximum on or about 1921 August 31. Adopting Turner's suggested period of $41^{d}\cdot5$ (*M.N.*, 82, 119), we should expect a maximum about 1922 June 17, which would be quite consistent with the observations given above.

Nova Lacertæ (1910).

Observed on 11 nights between May 14 and October 12. The observations confirm those of 1921 in showing that there is no appreciable variability in the light of this nova. The magnitude has been estimated at 14·1 on every occasion except one, when it was within $0^{m}\cdot1$ of this value.

Nova Persei (1901).

Observations began on 1922 July 15 and are still being continued. Up to the end of the year the star had been observed on 66 nights. Variability has again been very marked, but it is of such an apparently irregular type that it is not yet possible to say how far the observed fluctuations are truly periodic. During the period so far covered by the observations, the maximum and minimum magnitudes were respectively 12·22 and 14·07. It is intended to wait until April (when the nova ceases to be observable) before presenting the details of the observations.

Note on the Photography of Meteors. By F. A. Lindemann, F.R.S., and G. M. B. Dobson, M.A. (Plate 7.)

In another paper * we have shown how the temperature and density of the air at great heights may be calculated from the characteristics of meteors. As eye observations cannot be very accurate (especially as regards velocity, which is an important quantity for the calculation of the density of the air), photographic observations were undertaken to obtain accurate data.

The initial question to be settled was the choice of the best make of plate and best development. In order to bring out exceedingly faint

* *Proc. Roy. Soc.*, A, 102 (1922), p. 411.

images, it was found best to use a strong developer, and to develop for a very long time. This being so, it was necessary to select a rapid plate which would not fog badly with long development. After trying about a dozen different makes of plates, and a large number of different developers and concentrations, it was found that very good results were obtained with Gevaert plates (H and D 500) developed for thirty minutes in a tank with standard pyro-soda developer at about 15° C.

The well-known effect that the image caused by a faint light is brought up better when the plate is *slightly* fogged all over, was very markedly seen in these trial experiments. Tests were therefore made to determine the optimum amount of "fogging," by exposing successive strips across the plate to a faint light for various periods, and then causing the image of a star to travel uniformly along the plate. It was found that the star track came out clearest when the fogging could just be seen. It was found, however, that with a plate exposed for an hour or so, in our cameras with F 3·5 lenses, the stray light gave about the right amount of fogging. This may not necessarily be the case with telescopic photographs of stars, where slight fogging might improve the result.

To obtain a measurement of the heights of the meteors, two similar cameras were set up five km. apart pointing to the zenith, and oriented so that the edges of the plates were parallel to the base line. No exact setting or reference lines are necessary, as, of course, the star tracks afford ideal reference lines. Two cameras, with Goerz 25 cm. F/3·5 lenses, were kindly lent for the work by the Air Ministry, and proved very satisfactory. Each plate could be exposed for two hours without too much confusion of star tracks, etc.

To avoid the necessity of changing the plates by hand throughout the night, an automatic changing device was made, which is controlled by electric contacts in a clock. Fortunately the cameras were fitted with Goerz changing-boxes, in which the box has merely to be pulled out and pushed back to change the plate. This made mechanical changing very easy. Fig. 1 shows the apparatus diagrammatically. A is a gramophone motor, to the main spindle of which is fixed a crank B. By means of a connecting-rod this crank operates the changing-box (C) of the camera, which is shown "end on," looking at the lens. The governor of the motor is fitted with an extra brake (D) which will normally stop the motor. An electro-magnet releases this brake when contact is made by the clock. A cam (E) holds the brake off the governor, except for one position of the crank B, viz. when the changing-box is "home." Thus, as the clock makes contact for about five seconds every hour (or every two hours), the crank (B) makes one revolution and then stops for an hour, having changed one plate. It must then wait until the electro-magnet again releases the brake, when another plate is changed, and so on.

In order to be able to open and shut up the camera at any pre-arranged time during the night, the wheel F of the focal-plane shutter is provided with two stops, which engage with the armatures of two electro-magnets. The camera is set with the first stop in engagement, so that the shutter is closed; on this being released by a contact (J)

(which is closed each time the crank B revolves), the shutter opens. The second electro-magnet causes it to close. In order to be able to take any desired number of plates before closing the shutter, this second electro-magnet is operated as follows:—On the main spindle of the motor are a number of loose rings (G) arranged so that the first is free to turn one revolution with respect to the main spindle, but no more. The second ring is free to make one revolution with reference to the first ring, and so on. Each ring carries a stud which engages with a contact H, which closes the shutter. This contact can be set so that it is operated by the stud in any ring as desired. Now if set to engage the first ring, the spindle will turn once, and open the camera by the

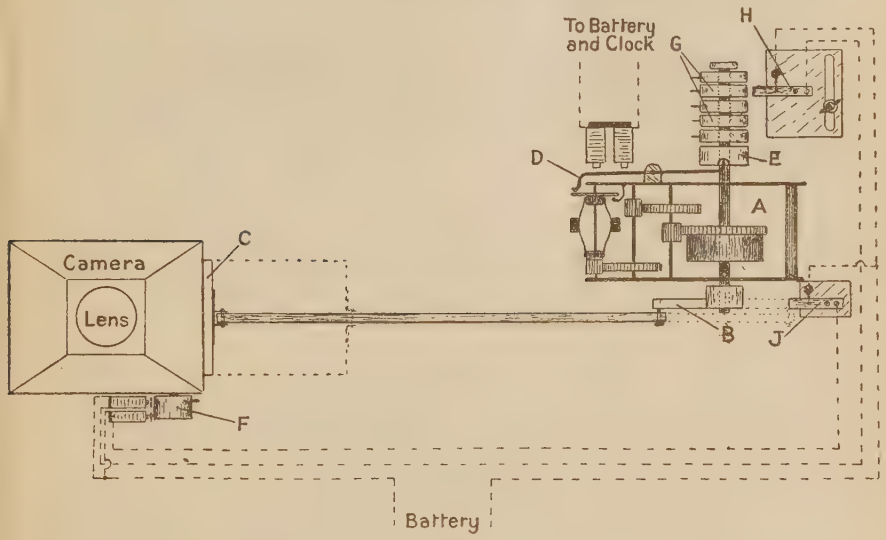


FIG. 1.

first contact, but the ring operating the second contact is still free. The second time the spindle turns, this ring is carried round, and causes the shutter to close. In the same way if the second contact be set to engage with the second, third, fourth, etc., ring, 2, 3, 4, etc., plates will be exposed before the shutter is closed. At the same time that the shutter is closed, a catch is also released and allows a weight to shut the sliding panel in the top of the changing-box, so that the whole is quite light-proof, even if left for several hours in sunshine during the early mornings in summer. The clock controlling the apparatus has two contacts in series, one which is made for about five seconds every hour, and another which is made as soon as the alarm is released. Thus the apparatus can be made to start at any time throughout the night (to allow for the moon), and to shut up at any time.

To obtain the speed of the meteor, a revolving sector is fixed over the lens of one camera. This sector is driven at a constant speed by

being connected to an electric motor through a special governor. By this means the track of a meteor is seen to be broken at regular intervals, and thus the velocity can be obtained.

Trouble was at first experienced by the lens fogging on cold nights, but this was prevented by shielding it as much as possible by a wooden hood, so that only just the necessary area of sky was left open.

Figs. 2 and 3 show the two photos of an unusually bright meteor, about midnight 1912 May 21 and 22, and fig. 4 shows the two plates combined. (Here one image is necessarily slightly out of focus in the reproduction.) The displacement of the image which gives the height must be measured parallel to the base line, *i.e.* up and down, in fig. 4. The actual images are extremely fine and can be measured with considerable accuracy. In the case of this meteor, the track was so long that neither the beginning nor end is on the plate. The height when it passed overhead was $83\frac{1}{2}$ km. and its velocity $35\frac{1}{2}$ km./sec. In most of the other meteors, both the beginning and end are on the plate. All the values obtained are about the average values used in the paper referred to, which were taken from eye observations.

Summary.

Apparatus is described which has been used to photograph meteors in order to ascertain their height and velocity, and the most suitable plates and methods of development for this purpose are considered.

Lines of Unknown Origin in Various Celestial Spectra. By F. E. Baxandall, A.R.C.Sc.

(*Communicated by the Director, Solar Physics Observatory, Cambridge.*)

In 1910, in a publication by the "Solar Physics Committee,"* a list was given of fairly prominent lines in celestial spectra for which no satisfactory origin had been found by reference to terrestrial spectra. The records drawn on in obtaining the list were Rowland's "Tables of Solar Wave-Lengths," Campbell's on the spectra of Wolf-Rayet stars and nebulæ, the Kensington publications on the chromosphere (supplemented by Dyson's) and corona, Pickering's lists for ζ Puppis and H.P. 1311, and the various records—published and unpublished—of stellar spectra obtained at South Kensington.

Since the publication of that list some of the lines have been traced to their terrestrial equivalents. On the other hand, in various spectroscopic records issued since 1910, other lines occur for which the terrestrial analogues have not been found. It has been thought desirable to bring the previous list up to date by discarding the identified lines and including the new unidentified lines. This has been done, and the lines still unidentified are given in the list which follows.

* *Lines of Unknown Origin in Various Celestial Spectra*, p. 71, 1910.

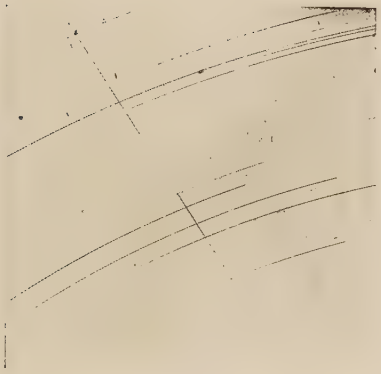


Fig. 2.

Fig. 3.

Fig. 4.

To keep the list within reasonable bounds, only the stronger unidentified lines in the individual spectra to which they belong have been included. For instance, in the solar spectrum (Fraunhofer lines) only those lines to which Rowland ascribes an intensity of four and upwards have been inserted.

For the nebular and Wolf-Rayet lines Wright's wave-lengths, as given in the Lick Observatory Publications, have been used. As representative of Wolf-Rayet spectra, Wright's record for B.D. + 30°.3639 (nucleus) has been used, as nearly all the lines observed in various stars of that type appear to be represented in the nucleus of this star.

The inclusion of bright lines of *o* Ceti in the table calls for some remark. Most of these have been provisionally identified, by the spectroscopists who have observed them, with low-temperature lines of such elements as Fe, Mg, Si, the identity being chiefly based on the apparently close agreements in wave-length. But the fact that there are many other iron lines belonging to the same temperature-classes which have not been recorded as bright lines in *o* Ceti, suffices to show that the apparent agreements in wave-length may be fortuitous and the supposed identifications fictitious. Until it can be shown that the lines involved in the identifications have, under certain laboratory conditions, some special behaviour which differentiates them from the rest of the lines belonging to the same elements, the identifications cannot be accepted without reserve. The only other bright lines occurring in *o* Ceti belong to hydrogen, and it is quite possible that the remaining lines may be due to some unknown gas. For the purpose of the present compilation the lines have therefore been considered as of unknown origin. The actual wave-lengths of the *o* Ceti lines as recorded by their observers have been here corrected for the mean velocity shift calculated from the known bright hydrogen lines. The assumption that all the bright *o* Ceti lines are subjected to the same velocity shift may be an arbitrary one, but to get at the probable normal wave-lengths of the strange bright lines it would seem to be the only justifiable procedure.

In some cases where the wave-lengths differ very little from each other, it is quite probable that the numbers given may be slightly different wave-length determinations of the same line occurring in different spectra. In such cases the wave-lengths have been bracketed together.

The classes under the headings indicate the particular celestial spectrum to which each line belongs.

The letters in brackets at the headings to the tabular columns indicate the references (given at the end of the paper) to the publications drawn on in the compilation of the table.

The figures in brackets in front of the wave-lengths refer to the notes immediately following the table.

[Table

Lines of Unknown Origin in Various Celestial Spectra.

Wave-Length.	Fraunhofer Spectrum (Rowland). (*a*)	Chromospheric Spectrum. (*b*)	Corona. (*c*)	Nebulæ. (*d*)	Wolf-Rayet Star B.D.+30°3639 (nucleus). (*e*)	Nova Persei (Kensington). (*f*)	Nova Geminorum No. 2 (Adams and Pease). (*g*)	Nova Geminorum No. 2 (Kensington). (*h*)	Nova Aquilæ No. 3 (Lunt). (*i*)	η Carinæ. (*j*)	H.P. 1311 (*A.G.C.* 8631). (*k*)	ο Ceti (Bright Lines). (*l*)	α Canum Venaticorum (*m*)	α Cygni. (*n*)	ε Ursæ Majoris. (*o*)	α Andromedæ. (*p*)	θ Aurigæ. (*q*)	ε Orionis. (*r*)
3388	..	..	×	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..
3426·4	..	..	..	×	..	..	..	..	..	..	..	..	..	..	..	..	..	..
3445	..	..	..	×	..	..	..	..	..	..	..	..	..	..	..	..	..	..
3455	..	..	×	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..
3601·3	..	..	×	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..
3611	..	..	..	..	×	..	..	..	..	..	..	..	..	..	..	..	..	..
3726·30	..	..	..	×	..	..	..	..	..	..	..	..	..	..	..	..	..	..
3729·05	..	..	..	×	..	..	..	..	..	..	..	..	..	..	..	..	..	..
3852·36	..	..	..	..	..	..	..	..	..	..	..	×	..	..	..	..	..	..
3860·0	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	×	..
3860·1	..	..	..	..	..	..	..	..	..	..	..	..	..	×	..	..	..	..
3868	..	..	..	..	..	×	..	..	..	..	..	..	..	..	..	..	..	..
3868·8	..	..	..	×	..	..	..	..	..	..	..	..	..	..	..	..	..	..
3905·59	..	..	..	..	..	..	..	..	..	..	..	×	..	..	..	..	..	..
3911·4	..	..	..	..	..	..	..	..	..	..	..	..	..	..	×	..	..	..
3938·2	..	×	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..
3938·34	..	..	..	..	..	..	..	..	..	..	..	×	..	..	..	..	..	..
3938·55	×	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..
3938·6	..	..	..	..	..	..	..	..	..	..	..	..	..	×	×	..	..	..
3945·2	..	..	..	..	..	..	..	..	..	..	..	..	..	×	..	..	..	..
3952·3	..	×	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..
3954·3	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	×	..
3967·66	..	..	..	×	..	..	..	..	..	..	..	..	..	..	..	..	..	..
3967·73	..	..	..	..	..	..	..	..	..	..	..	×	..	..	..	..	..	..
(1) 3968·35	×	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..
(1) 3968·89	×	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..
3970·01	..	..	..	..	..	×	..	..	..	..	..	..	..	..	..	..	..	..
3984·0	..	..	..	..	..	..	..	..	..	..	..	..	×	..	..	..	..	..
3984·1	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	×	..	..
3987·0	..	..	×	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..
4006·97	..	..	..	..	..	..	..	..	..	..	..	×	..	..	..	..	..	..
4021·6	..	×	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..
4024·6	..	..	..	..	..	..	..	..	..	..	..	..	..	×	..	..	..	..
4038	..	..	..	..	..	..	..	..	..	..	..	..	×	..	..	..	..	..

Lines of Unknown Origin in Various Celestial Spectra—continued.

Wave-Length.	Fraunhofer Spectrum (Rowland).	Chromospheric Spectrum.	Corona.	Nebulæ.	Wolf-Rayet Star B.D.+30°3639 (nucleus).	Nova Persei (Kensington).	Nova Geminorum No. 2 (Adams and Pease).	Nova Geminorum No. 2 (Kensington).	Nova Aquilæ No. 3 (Lunt).	η Carinæ.	H.P. 1311 (*A.G.C.* 8631).	ο Ceti (Bright Lines).	α Canum Venaticorum.	α Cygni.	ε Ursæ Majoris.	α Andromedæ.	θ Aurigæ.	ε Orionis.
4056·7	..	..	..	..	×	..	..	..	..	..	..	..	..	..	..	..	..	..
(2) 4059·2	..	..	..	..	..	..	..	..	..	..	×	..	..	..	..	..	..	..
4068·77	..	..	..	×	..	..	..	..	..	..	..	..	..	..	..	..	..	..
4069·7	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	×
4070	..	..	..	..	×	..	..	..	..	..	..	..	..	..	..	..	..	..
4076	..	..	..	..	..	..	..	..	..	..	..	..	×	..	..	..	..	..
4076·3	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	×	..
4076·37	..	..	..	×	..	..	..	..	..	..	..	..	..	..	..	..	..	..
4086·7	..	×	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..
4122·8	..	..	..	..	..	..	..	..	..	..	..	..	..	..	×	..	..	..
4122·9	..	..	..	..	..	..	..	..	..	..	..	..	..	×	..	..	..	..
4123·0	..	×	..	..	..	..	..	..	..	..	..	..	×	..	..	..	..	..
4128·9	..	..	..	..	..	..	..	..	..	..	..	..	..	..	×	..	..	..
4132·4	..	..	..	..	..	..	..	..	..	..	..	..	×	..	..	..	..	..
4132·6	..	..	..	..	..	..	..	..	..	..	..	..	..	..	×	..	..	..
4137·5	..	×	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..
4155·4	..	..	..	..	×	..	..	..	..	..	..	..	..	..	..	..	..	..
4159·35	×	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..
4163·8	..	..	..	..	×	..	..	..	..	..	..	..	..	..	..	..	..	..
4184·16	×	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..
4188·89	×	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..
4191·8	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	×	..
4198·40	×	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..
4202·10	..	..	..	..	..	..	..	..	..	..	..	×	..	..	..	..	..	..
4215·90	..	..	..	..	..	..	..	..	..	..	..	×	..	..	..	..	..	..
4229·6	..	..	..	..	×	..	..	..	..	..	..	..	..	..	..	..	..	..
4231·3	..	..	×	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..
4233·31	..	..	..	..	..	..	..	..	..	..	..	×	..	..	..	..	..	..
4244·25	..	..	..	..	..	..	..	..	..	×	..	..	..	..	..	..	..	..
4258·6	..	..	..	..	..	..	..	..	..	..	..	..	..	×	..	..	..	..
4273·6	..	..	..	..	..	..	..	..	..	..	..	..	..	×	..	..	..	..
4275·6	..	..	..	..	..	..	..	..	..	..	..	..	..	..	×	..	..	..
4277·01	..	..	..	..	..	..	..	..	..	×	..	..	..	..	..	..	..	..
4287·45	..	..	..	..	..	..	..	..	..	×	..	..	..	..	..	..	..	..

Lines of Unknown Origin in Various Celestial Spectra—continued.

Wave-Length.	Fraunhofer Spectrum (Rowland).	Chromospheric Spectrum.	Corona.	Nebulæ.	Wolf-Rayet Star B.D.,+30°3639 (nucleus).	Nova Persei (Kensington).	Nova Geminorum No. 2 (Adams and Pease).	Nova Geminorum No. 2 (Kensington).	Nova Aquilæ No. 3 (Lunt).	η Carinæ.	H.P. 1311 (*A.G.C.* 8631).	o Ceti (Bright Lines).	α Canum Venaticorum.	α Cygni.	ε Ursæ Majoris.	α Andromedæ.	θ Aurigæ.	ε Orionis.
4291·64	..	..	..	..	..	..	..	..	..	..	..	×	..	..	..	..	..	..
4301·26	×	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..
4304·10	×	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..
4307·87	..	..	..	..	..	..	..	..	..	..	..	×	..	..	..	..	..	..
4325·8	..	..	..	..	×	..	..	..	..	..	..	..	..	..	..	..	..	..
4342·74	..	..	..	..	..	..	..	..	..	×	..	..	..	..	..	..	..	..
4352·30	..	..	..	..	..	..	..	..	..	..	..	×	..	..	..	..	..	..
4359·11	..	..	..	..	..	..	..	..	..	×	..	..	..	..	..	..	..	..
4359·5	..	..	×	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..
4363·37	..	..	..	×	..	..	..	..	..	..	..	..	..	..	..	..	..	..
4364	..	..	..	..	..	×	..	..	..	..	..	..	..	..	..	..	..	..
4375·96	..	..	..	..	..	..	..	..	..	..	..	×	..	..	..	..	..	..
4377·0	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	×	..
4414·04	..	..	..	..	..	..	..	..	..	×	..	..	..	..	..	..	..	..
(3) 4416·43	..	..	..	..	..	..	..	..	..	×	..	..	..	..	..	..	..	..
4427·42	..	..	..	..	..	..	..	..	..	..	..	×	..	..	..	..	..	..
4441·4	..	..	..	..	×	..	..	..	..	..	..	..	..	..	..	..	..	..
4446·55	..	×	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..
4448	..	..	..	..	..	..	..	..	..	..	..	..	×	..	..	..	..	..
4448·30	..	..	..	..	..	..	..	..	..	..	..	×	..	..	..	..	..	..
4452·11	..	..	..	..	..	..	..	..	..	×	..	..	..	..	..	..	..	..
4457	..	..	..	..	×	..	..	..	..	..	..	..	..	..	..	..	..	..
4457·97	..	..	..	..	..	..	..	..	..	×	..	..	..	..	..	..	..	..
4461·89	..	..	..	..	..	..	..	..	..	..	..	×	..	..	..	..	..	..
4474·87	..	..	..	..	..	..	..	..	..	×	..	..	..	..	..	..	..	..
4516·6	..	..	..	..	×	..	..	..	..	..	..	..	..	..	..	..	..	..
4555	..	..	..	..	×	..	..	..	..	..	..	..	..	..	..	..	..	..
4567	..	..	×	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..
4570·94	..	..	..	..	..	..	..	..	..	..	..	×	..	..	..	..	..	..
4603·9	..	..	..	..	..	..	..	..	×	..	..	..	..	..	..	..	..	..
(4) 4608·3	..	..	..	..	..	..	..	..	..	..	×	..	..	..	..	..	..	..
(4) 4610	..	..	..	..	..	..	×	..	..	..	..	..	..	..	..	..	..	..
(5) 4636	..	..	..	..	..	..	×	..	..	..	..	..	..	..	..	..	..	..
(5) 4639·2	..	..	..	..	..	..	..	×	..	..	..	..	..	..	..	..	..	..

Lines of Unknown Origin in Various Celestial Spectra—continued.

Wave-Length.	Fraunhofer Spectrum (Rowland).	Chromospheric Spectrum.	Corona.	Nebulæ.	Wolf-Rayet Star B.D. $+30°.3639$ (nucleus).	Nova Persei (Kensington).	Nova Geminorum No. 2 (Adams and Pease).	Nova Geminorum No. 2 (Kensington).	Nova Aquilæ No. 3 (Lunt).	η Carinæ.	H.P. 1311 (*A.G.C.* 8631).	o Ceti (Bright Lines).	α Canum Venaticorum.	α Cygni.	ϵ Ursæ Majoris.	α Andromedæ.	θ Aurigæ.	ϵ Orionis.
4654·6	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	×
4666	..	..	..	..	×	..	..	..	..	..	..	..	..	..	..	..	..	..
4699·51	×	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..
4700·34	×	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..
4711·6	..	..	..	×	..	..	..	..	..	..	..	..	..	..	..	..	..	..
4720	..	..	..	..	..	×	..	..	..	..	..	..	..	..	..	..	..	..
4728·35	..	..	..	..	..	..	..	..	..	×	..	..	..	..	..	..	..	..
4731·4	..	×	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..
4731·7	..	..	..	..	..	..	..	..	..	..	..	..	..	×	..	..	..	..
4740·4	..	..	..	×	..	..	..	..	..	..	..	..	..	..	..	..	..	..
4757·5	..	×	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..
4762·0	..	×	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..
4786·4	..	..	..	..	×	..	..	..	..	..	..	..	..	..	..	..	..	..
4811·0	..	×	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..
4814·78	..	..	..	..	..	..	..	..	..	×	..	..	..	..	..	..	..	..
4815	..	..	..	..	..	..	..	..	..	..	..	..	×	..	..	..	..	..
4889·64	..	..	..	..	..	..	..	..	..	×	..	..	..	..	..	..	..	..
4959·09	..	..	..	×	..	..	..	..	..	..	..	..	..	..	..	..	..	..
5007·02	..	..	..	×	..	..	..	..	..	..	..	..	..	..	..	..	..	..
5130·9	..	..	..	..	×	..	..	..	..	..	..	..	..	..	..	..	..	..
5251·1	..	..	..	..	×	..	..	..	..	..	..	..	..	..	..	..	..	..
5303·7	..	..	×	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..
5755·0	..	..	..	×	..	..	..	..	..	..	..	..	..	..	..	..	..	..
6302	..	..	..	×	..	..	..	..	..	..	..	..	..	..	..	..	..	..
6374·2	..	..	×	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..
6548·3	..	..	..	×	..	..	..	..	..	..	..	..	..	..	..	..	..	..
(6) 6583·8	..	..	..	×	..	..	..	..	..	..	..	..	..	..	..	..	..	..

As some of the lines in the preceding list call for more mention than could be conveniently included in a "Remarks" column, special notes have been made concerning them, and these now follow :—

Note (1). Doubtfully genuine lines. Rowland describes them in his "Corrections" as "mostly shading of H."

Note (2). The Harvard observers describe this as double, the more refrangible portion being wider and fainter than the other.

Note (3). Probably involves the strongly enhanced iron line 4417·00, as all the other prominent enhanced lines of iron occur in η Carinæ.

Note (4). There is rather a large difference in the wave-lengths of these two lines as recorded, but it is not at all improbable that the lines in the individual spectra are identical in origin. The Harvard observers variously record the line in the spectrum of H.P. 1311 (*A.G.C.* 8631) as 4608·3 and 4606, but the former is probably the more correct wave-length. The line is broad and hazy, and it is consequently difficult to estimate its true position. There is a line of oxygen at λ4609·40 which, in laboratory spectra, appears to have a differential behaviour as compared with other oxygen lines. There is thus suggestive evidence that the laboratory and stellar lines might be identical, but this identity cannot be considered as established.

Note (5). These two lines, occurring individually in different records of the lines of Nova Geminorum, when that star was in its later stages, may be identical, although the difference in wave-length is rather large. The line, however, is not an easy one to measure, being somewhat broad and diffuse.

Note (6). There is a strong carbon line (6583·0) very near the position of this nebular line, but, as the neighbouring carbon line 6577·5 does not appear in the nebular spectra, the close agreement in wave-length referred to is probably fortuitous. This nebular line and its strong companion at λ6548 are probably of gaseous origin like the great majority of the other nebular lines whose terrestrial analogues have been definitely traced.

The following lines, which were included in the previous list of unidentified lines in the Solar Physics Committee's publication of 1910, have since been traced in terrestrial spectra, and have therefore been discarded from the category of "unknown" lines:—

λ.	Origin to which traced.	Celestial Spectrum occurring in	Remarks.
3943·9	pMn	α Andromedæ	Note (*a*)
3986·90	Mg	Solar (Fraunhofer)	,, (*b*)
4026·0	pHe	ζ Puppis, etc.	,, (*c*)
4057·67	Mg	Solar (Fraunhofer)	,, (*b*)
4137·0	pMn	α Andromedæ	,, (*a*)
4161·68	pTi	Solar (Fraunhofer)	,, (*d*)
4167·48	Mg	Solar (Fraunhofer and Chromosphere)	,, (*b*)
4200·7	pHe	ζ Puppis HP 1311, etc.	,, (*c*)
4206·3	pMn	α Andromedæ	,, (*a*)
4282·4	pMn	,, ,,	,, (*a*)
4462·3	Fe Mn	Chromosphere	,, (*e*)
4496·8	Cr	,,	,, (*f*)
4542·4 4544	pHe	Early B Stars O Type Stars, etc.	,, (*c*)
4600·8	Cr	Chromosphere	,, (*g*)
4650 4652	pC	Wolf-Rayet Stars Nova Persei, etc.	,, (*h*)
4687 4688	pHe	Nebulæ. Wolf-Rayet Stars, etc.	,, (*i*)

Notes relating to preceding Table.

Note (*a*). These lines in the spectrum of α Andromedæ were identified by Baxandall* as enhanced lines of manganese, the enhanced lines of that element being specially prominent in this particular stellar spectrum.

Note (*b*). These strongly marked lines in the Fraunhofer spectrum have been traced by Fowler to magnesium.† Rowland's failure to identify them with magnesium lines was probably due to the fact that, whereas in the Fraunhofer spectrum they are sharp, these magnesium lines are generally considerably winged, and the apparent middles of the lines in laboratory spectra do not represent the true wave-lengths.

Note (*c*). These lines, first noted in ζ Puppis by Pickering, have been traced in laboratory spectra by Fowler,‡ and the conclusion he has arrived at is that they are due to helium in some form or other. They were formerly considered by Pickering and others to be due to a modified form of hydrogen.

Note (*d*). This line is identical in position with an enhanced line of titanium. Judging from the Fraunhofer intensity of other titanium lines, however, the intensity of this solar line seems rather high to be ascribed to titanium alone. It may be that under the solar conditions this particular titanium line is specially enhanced, and at any rate it seems best to discard the lines from a list of unidentified lines.

Note (*e*). According to Mitchell § the chromospheric line nearest to this is identical with Rowland's solar line 4462·165 (Fe Mn), and the line given by Lockyer at 4462·3 is probably the same.

Note (*f*). In the eclipse record published by Mitchell ‖ he gives a line at 4497·11 (3) which he ascribes to Zr. It is very probable, however, that this eclipse line is identical with the stronger adjacent solar line 4497·02 (Cr), especially as all the Cr lines of equal intensity to 4497·0 are found in Mitchell's record. Lockyer's line at 4496·8 is probably identical with Mitchell's.

Note (*g*) Mitchell ¶ gives this chromospheric line as 4600·93, and identifies it with Rowland's line 4600·93 (Cr). This is probably correct, but the chromospheric intensity seems high for the Cr line as compared with those of equally strong Cr arc lines.

Note (*h*). This Wolf-Rayet line has been provisionally identified by the Kensington observers ** with a double carbon line which had been found to be represented as a strong double absorption line in ε Orionis. More recently Merton has traced the identity of other Wolf-Rayet lines with high-temperature carbon lines, and as he also got the double line of carbon (and a third adjacent line) the identity of the 4650 stellar line with carbon has now been assumed.

In the previously published list of unknown lines 4650 was given as occurring also in the later stages of the Nova Persei spectrum (Kensington). This was an error, no line at that wave-length having been recorded in the Kensington publications.

Note (*i*). This line was first traced in a laboratory spectrum of helium at Kensington in 1898, and its probable identity with an important eclipse line was pointed out in a publication †† of the Viziadrug eclipse results. In later researches Fowler has obtained many laboratory spectra showing the line, and has discussed in much detail its relation to the series of helium lines the celestial equivalents of which were first found by Pickering in the spectrum of ζ Puppis. There can be no doubt that the line in question accounts for the prominent line in various celestial spectra such as Wolf-Rayet stars, nebulæ, and novæ in their later stages.

* *M.N.R.A.S.*, **74**, 252.
‡ *Proc. Roy. Soc.*, **73**, 62.
‖ *Ibid.*, 456.
** *Proc. Roy. Soc.*, **82**, 534.
† *Proc. Roy. Soc.*, **89**, 133–137.
§ *Ast. Phys. Jour.*, **38**, 454.
¶ *Ibid.*, 457.
†† *Phil. Trans.*, **197**, 202, 1901.

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(See headings to Table.)

(*a*) Rowland's Solar Tables.
(*b*) 1. *Kensington Phil. Trans.*, **197**, 208, 1901.
2. Dyson, *Phil. Trans.*, **206A**, 438, 1906.
(*c*) Campbell and Moore, *Lick Obs. Bull.*, **10**, 10.
(*d*) Wright, *Pub. Lick Obs.*, **13**, Table 11, following p. 242.
(*e*) Wright, *Pub. Lick Obs.*, **13**, 22.
(*f*) Lockyer, *Proc. Roy. Soc.*, **69**, 135.
(*g*) Adams and Pease, *Ast. Phys. Journ.*, **40**, 296.
(*h*) Publication by Solar Physics Committee, "Phenomena of New Stars," p. 32, 1914.
(*i*) Lunt, *M.N.R.A.S.*, **80**, 530.
(*j*) Moore and Sanford, *Lick Obs. Bull.*, **8**, 59.
(*k*) Miss Cannon, *Ann. Harv. Coll. Obs.*, **28**, Pt. II., 248.
(*l*) 1. Stebbins, *Lick Obs. Bull.*, **2**, 91.
2. Adams and Joy, *Pub. Ast. Soc. Pac.*, **30**, 193.
(*m*) Belopolsky, *Ast. Nach.*, **196**, No. 4681, 1.
(*n*) Publication by Solar Physics Committee, "Catalogue of 470 of the Brighter Stars," p. 30, 1902.
(*o*) Publication by Solar Physics Committee, "Researches on the Chemical Origin of Various Lines in Solar and Stellar Spectra," p. 31, 1910.
(*p*) Lockyer and Baxandall, *Proc. Roy. Soc.*, **77**, 550.
(*q*) Lockyer and Baxandall, *Proc. Roy. Soc.*, **77**, 550.
(*r*) Publication by Solar Physics Committee, "Comparison of the Spectra of Rigelian, Crucian, and Alnitamian Stars," p. 6, 1914.

On the Estimation of the Continuous Spectrum of Stars.

By R. A. Sampson, F.R.S. (Plates 8, 9, 10).

The following pages give a preliminary account of an investigation which has been in progress for some years at the Royal Observatory, Edinburgh, on the use of the photoelectric cell in Astronomy. It is unnecessary to describe this cell here beyond saying that in its simplest form it consists of an evacuated bulb having on its inner face a deposit of metallic potassium. A collecting rod or grid passes through the wall of the bulb, and the terminals of a battery charge the face negatively and the grid positively. The bulb is thus a condenser. But if light falls on the deposit, it causes the release of electrons which make their way to the positively charged grid; so that under the action of light the bulb behaves as a valve or unilateral resistance of varying strength, and can thus be employed by suitable calibration as a measure of the incident light.

With two such bulbs arranged in cascade, the coat of A being connected to the negative terminal of the battery and the grid of B to the positive terminal, while the grid of A is connected to the coat of B by a bridge C, we have a balancing arrangement, such that if light from a lamp falls on B, a negative charge passes to the grid of B, and C is left with a positive charge, but if it falls on A, the electrons emitted from A reach C and confer on it a negative charge.

In the astronomical applications hitherto published, the single bulb has been used directly at the telescope as a species of eye. It has been proved to be a method admitting refinement of measurement far surpassing any other, in the problems to which it has been made applicable.* At the same time it adds difficulties of its own to the already rather confused field of photometry. Besides the physical and mechanical difficulties of keeping the electrical state of the cell constant when attached to a telescope in different pointings, the most essential questions of photometry are those which distinguish radiations of different wave-lengths. A method of so much refinement could not be satisfied permanently with a simple "magnitude" measure, in which lights of all colours were integrated together. But to calibrate the cell in respect to its behaviour to different lights would be a serious complication. These considerations become important just in proportion to the refinement of the method. There is the further consideration that

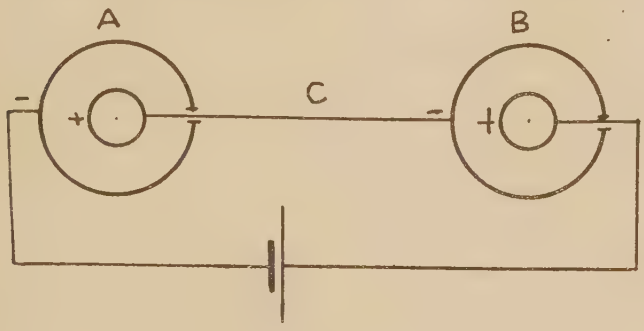


Fig. 1.

visual work, with which it must be classed, hardly uses the observer's time to the best advantage, whereas photography allows him to supplement in many ways the time he spends at the telescope. Considering these points, it was decided to obtain first photographic records for which at any rate the difficulties were familiar, and then to estimate these with the photoelectric cell used in the form of the Koch microphotometer.† The arrangement of this photometer is shown in figure 2 by the usual conventional diagram. C_1C_2 are the sensitive coats of the two cells, G_1G_2 the collecting grids. C_1 is joined to the negative pole of a high tension battery the positive pole of which is earthed. G_2 is connected to Earth, while G_1 is joined to C_2 and to the insulated thread between the plates of a string electrometer. The same lamp L illuminates C_2, passing through an adjustable diaphragm D, and C_1, passing through the photographic deposit on a plate P which it is required to estimate. In principle the action is that of the two balancing cells, though the actual explanation is intricate, because the cells are filled

* *Cf.* Elster and Geitel, *Phys. Zeitschr.*, **12** (1911), 609, and **13** (1912), 739. Stebbins, *Ap. J.*, **51** (1920), 193; **53** (1921), 105. Lindemann, A. F. and F. A., *M.N.*, **79** (1919), 343.

† Koch, *Ann. d. Phys.*, **39** (1912), 705.

with a small quantity of helium and because of other details. But it is not necessary to pursue these. By using the balancing effect, the density of any photographic deposit may be compared with a standard. Provided the same cells and thread are kept, and the battery potentials are constant, and the electric circumstances of insulation, etc., satisfactory, the scale of the record will be unchanged. The lamp and its variations are almost entirely eliminated.

At the time the research began, it would have been impossible to purchase any part of the instrument, and it is not very easy to make one from description. It is highly to the credit of Mr. E. A. Baker that it has succeeded. He learnt to make the cells, and constructed first an

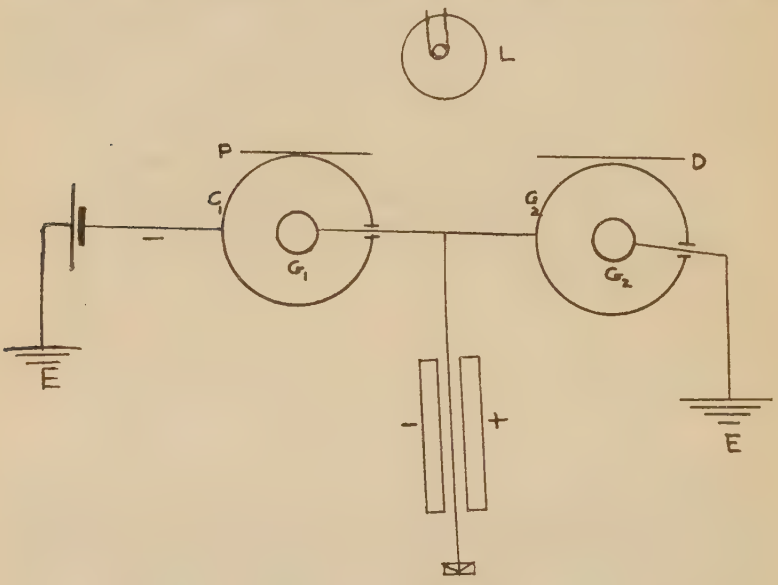


FIG. 2.

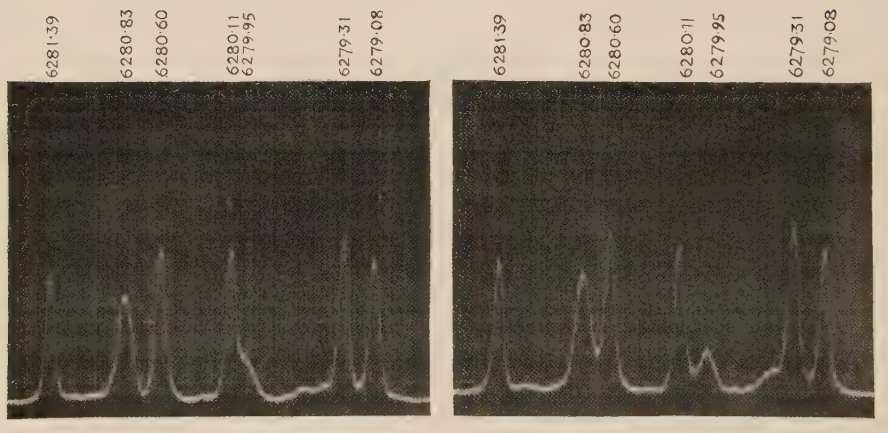
instrument where the deflection of the thread, the best form for which proved to be a Wollaston wire, was read visually. This was used for some time chiefly to measure magnitudes of stellar images of the same fields photographed through different light filters. As this showed promise of complete success, a second instrument was made specially for estimating spectra, which was provided with projection apparatus for the thread, and a camera, moving the photographic paper past an aperture, in agreement with the movement of the spectral plate under estimation past a slit, through which the light of the lamp fell upon the sensitive coat of the cell. The Observatory is fortunate in possessing many first-rate pieces of physical apparatus not in immediate use. Among these were found a most suitable camera, and a measuring machine with fine screw and all necessary adjustments. Very little except some gear and wheelwork required to be made outside the Observatory. In this form

SPECTRA OF SUN'S LIMBS

3rd Order x 21

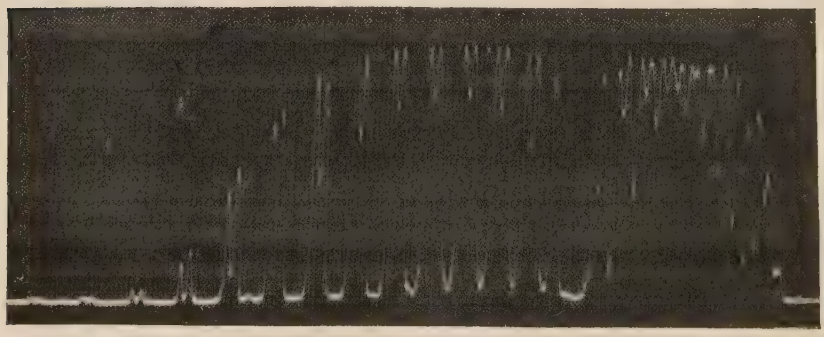
W. LIMB

E. LIMB



SOLAR SPECTRUM [A] BAND

1st Order x 5



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it is completely equivalent to Koch's photometer. The instrument is steady in behaviour and very satisfactory in use. Its management has been in Mr. Baker's sole charge.

The result of passing a spectrum past the slit of this instrument is to derive a new version of that spectrum in two dimensions, the density of the deposit being given by the ordinates, just as in Langley's bolograms of solar radiation. Absorption or bright lines are shown by serrations. The strength of the stellar emission in different wave-lengths is shown by the general rise and fall of the curve, subject of course to the modifications which are introduced by atmospheric and instrumental absorption and the varying inertia of the photographic plate. The scale of the conversion of the spectrum into these photograms, as I call them, can be varied by changing the gear. The apparatus being automatic and tireless, reproduces the original record with complete fidelity, and brings out with definiteness features which no eye could be sure of. It can be used in this way for very delicate and difficult measures of position, just as Koch used it for examining detail of structure of spectral lines. As illustrations I give in Plate 8 (i) a large scale photogram of a portion of the solar spectrum taken in the third order, near $\lambda 6280$, showing the Doppler displacement of solar lines relative to terrestrial lines on the Sun's west and east limbs, magnified $\times 21$; and (ii) the (A) band of the spectrum, taken in the first order, and magnified $\times 5$.*

Before the instrument was proved, it was feared that pin-holes, specks of dust, etc., would vitiate the record, and that a considerable proportion of spurious detail would be shown. It appears, however, that there is almost none. When any such anomaly is present, it stands out in a way that declares its character at once. Thus in the spectrum of Capella, from a negative of about one inch in length, between (C) and (K), the photogram shows upwards of 120 clearly marked details that can be identified without uncertainty in every trace. Of course it is only conventionally that these details can be called "lines." With so small an original scale each "line" must really be an extensive composite band. But it appears that wherever any outstanding line is present in the spectrum, for example, $\lambda 4216$, which is marked as intensity Fe 2, Sr 5, on Rowland's scale for the solar spectrum, or even considerably fainter ones, a characteristic serration appears in the photogram.

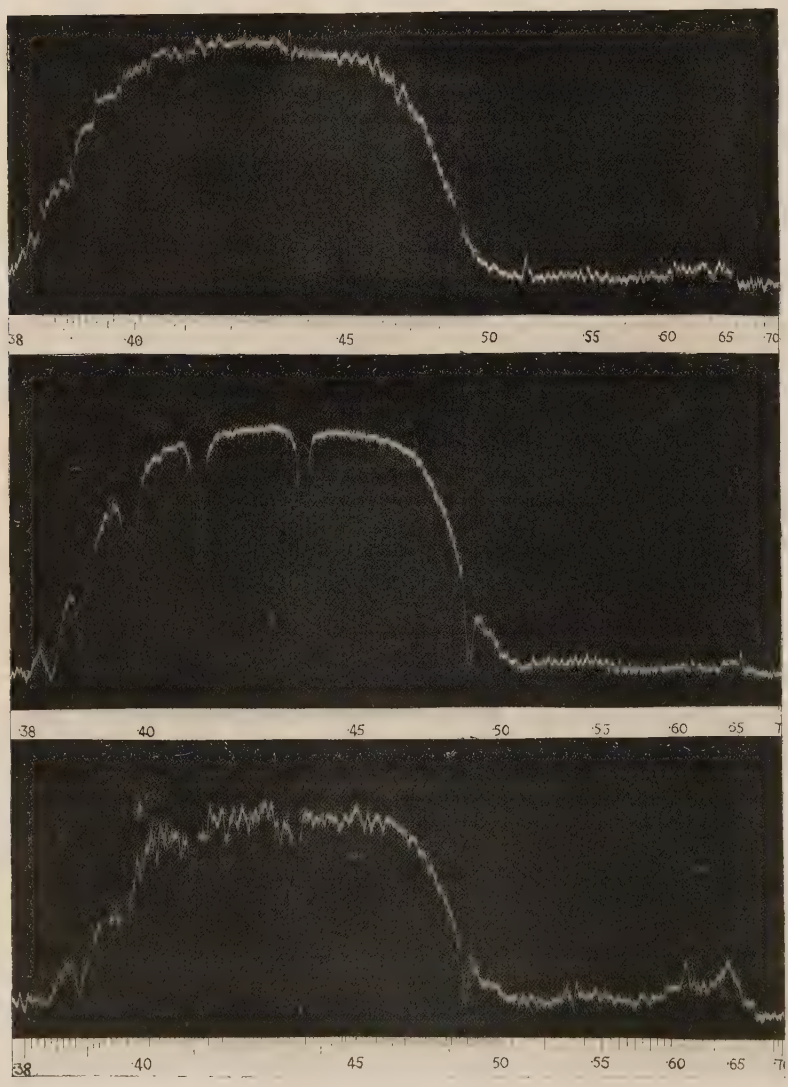
Some specimens of photograms of the spectra of characteristic stars are shown in Plates 9, 10. It should be recalled that these are preliminary results which I hope experience will soon surpass. Even so, it will hardly be contested that for the purposes of character and identification, of classifying and committing to memory, they are much better than ordinary spectra, even were these most carefully enlarged in reproduction. It is not an exaggerated summary to say that they add a new dimension to the spectrum, which, so soon as it is brought into a reliable state as regards measurement, should prove of similar importance to the measures of position offered by the lines. Evidently they will lend themselves to a variety of purposes, especially where it is required to get the maximum of definite measurement from hazy material without introducing any

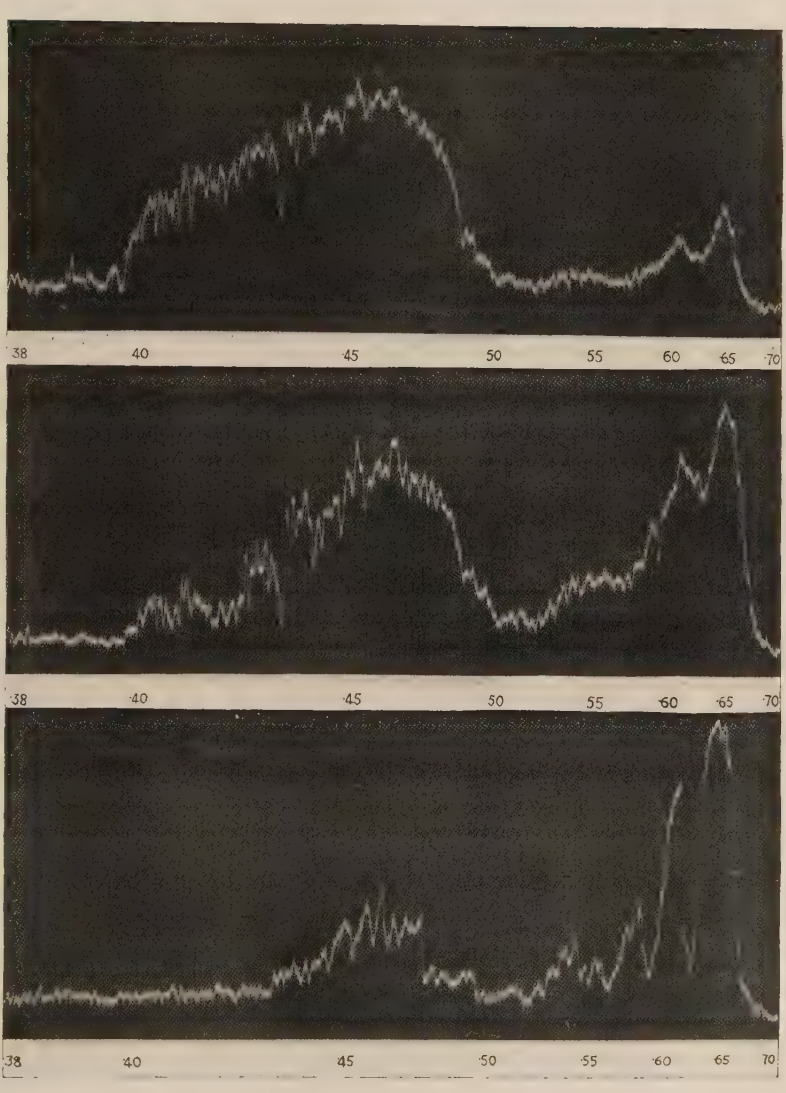
* The reproductions are reduced $\times \frac{2}{3}$.

personality of judgment—a common and important requirement. They also invite comprehensive discussion of the relative intensities of lines and their bearing on the strata of a star's atmosphere. It appears to me that the instrument may ultimately prove as important an aid to astronomy as the micrometer itself. But in what follows I shall confine myself to a preliminary consideration of a single problem, namely, to estimate the intensity of continuous radiation from stars of different spectral class over a considerable range of the spectrum.

This, of course, is no unfamiliar idea. It is akin to the measurement of intensities of lines for present-day classification, and for the determinations of absolute magnitude. Gill insisted on the point at the Astrographic Congress of 1909,* requiring a scientific definition of scale of photographic magnitudes, which should take account—(1) of the relative intensity of radiations of different wave-lengths in the stars studied, (2) of the radiations to which the photographic plate was most sensitive. Spectra illustrating the idea, taken by the wedge method, but not differing in principle from photograms, are shown in the Draper Catalogue, 12^h–14^h, Frontispiece.† It has been applied by H. H. Plaskett ‡ to assign the temperature of γ Cassiopeiæ, and in an extensive investigation at Potsdam by Wilsing and others,§ with the 80-cm. refractor, to find the temperatures of 199 stars, by visual comparison of their spectra at a number of different wave-lengths with the radiations of an artificial black body. But I would remark that a variety of method is desirable. In spite of the experience and skill of the Potsdam observers, their method seems still open to large errors, in some cases amounting to a full reversal of the order of development. Thus to β Cas. and α Per., both F5 in class, are assigned temperatures of 5200° and 5700° respectively, while Capella, Go, shows 7100°. A mere glance at the photograms of the first and third, which are shown, will suffice to show that the former places its maximum far further towards the blue than the latter. It is not to be expected that the problem of reading and interpreting the intensities of stellar spectra should be brought into an established state at a single step. It will be recalled that the early measures of the Doppler displacement of lines were completely unreliable in magnitude and in sign. It will be seen that the following pages might be called a discussion rather than a solution. Apart from the fact that they involve calculations of photographic sensitometry which are in some respects new, and may therefore, I suppose, be disputed, their interpretation as stellar temperatures immediately raises the question—Temperatures at what level? The temperature of the Sun, at 93 million miles from its centre, calculated according to principles of radiative equilibrium, is about 60° F or 289° A.‖ Even in the case of the Sun, keeping to its visible disc, its radiation at the centre shows a much more pronounced peak than equivalent radia-

* *Carte du Ciel—Comité International, Conférence de* 1909 (43). Paris.
† *Harvard Annals*, 95 (1920).
‡ H. H. Plaskett, *M.N.*, 80 (1920), 771.
§ *Potsdam Pub. Astrophys. Obs.*, No. 74, Bd. 24 (1) (1919).
‖ *Cf.* Poynting, *Ph. Tr.*, 202 (1903), 525, who refers to Wien, *Congrès International de Physique*, ii. 30.


γ Cas., Bop.
Vega, Ao.
β Cas., F 5.
38
40
45
50
55
60
65
70


Capella, Go.
α Cas., Ko.
β Peg., M3.
·38
·40
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·55
·60
·65
·70

tion from a black body, while over its disc, proceeding from centre to limb, there is a progressive reduction of intensity, due no doubt to general absorption in its "atmosphere," which increases as the wave-length diminishes. If the Sun's radiations from its whole disc are integrated, as must be done for a star, the general effect of this absorption would make the Sun appear sensibly redder, and if we carry the same allowance to another star, transforming it only to different wave-lengths, by Wien's law $\lambda T = \text{constant}$ for black-body radiation, a blue star would also appear redder but in less degree. Hence integrated measures of stellar radiation would exaggerate actual temperature differences and will require to be allowed for. But even though their spectra are closely similar, what confidence can we have in carrying over an allowance derived from a dwarf star like the Sun to a giant star like Capella, with a luminosity 75 times as great, and a density presumably as low as those found for giant spectroscopic binaries of the same spectral class.* Again, are we justified in supposing that the matter is so simple as a continuous sequence of black-body radiators? If there are blue, white, yellow, and red stars, why are there no green stars? In fact the problem is left with a demand for the next step—an inquiry into stellar atmospheres, the importance of which upon other grounds Mr. E. A. Milne has recently urged.†

I now come to particulars of the material here presented. The telescope employed is a 6-inch Cooke photovisual, of focal length about 100 inches, with a Merz objective prism of dense flint of 12° angle. With the plate-holder tilted at about 20°, spectra are obtained that are in sharp focus from (B) to (K). Recognisable lines are indeed obtained on some plates (No. 32_5, Regulus B6) down to H_ν ($\lambda 3712$) or beyond. The scale on the negative varies from 1 mm. = 280 AU at (C) to 1 mm. = 51 AU at (K), being overall 1 mm. = 121 AU. The clock of the telescope was not meant for photographic work, and gave some trouble, which was finally cured by allowing it to run a little fast and bringing the star back to trail several times along the horizontal wire of the guider between two vertical wires. For Polaris the telescope is simply clamped and the star allowed to trail once between these two wires.

The plates used are Ilford Panchromatic, developed with pyro-soda according to the instructions. They are generally excellent, but some batches have proved decidedly more sensitive in the red than others. The whole of the spectra discussed below were taken by Mr. J. Storey and are of very good quality; his patience and skill overcame one by one the numerous instrumental troubles.

The plan of the observation was the following. It was recognised that radiation of any specific wave-length was subject to selective change owing to the atmosphere, the instrument, and the photographic plate. In order to ensure that, as far as possible, measures made should be significant of stellar circumstances only, at least one trail of Polaris was taken on each plate, and the measures of the other stars recorded on the same plate are made relative to this. The theory by which the com-

* *Cf.* Shapley, *Ap. J.*, **42** (1901), 271.
† E. A. Milne, *Phil. Trans.*, **223** (1922), 201.

parison is made is given below. As to atmospheric absorption, that of course could not be controlled or determined, and a comparison with a record of Polaris separated by only a few minutes of time from the record of the star was an imperfect guarantee, the more so since they were taken in different parts of the sky. Light can, however, be thrown upon this by collecting all the photograms of Polaris and comparing them with one another, as is done below. The general conclusion from this examination is that the method of comparison with Polaris, as it is used, supplies a fully trustworthy standard. Of course large variations of intensity appear over the whole spectrum, and alter completely the scale of ordinates in the photogram, but associated with this change comes the question,—If we deliberately vary the intensity by taking, say, 5 and then 7 trails of Capella, can we so measure the photograms that they shall be equivalent? If this question can be answered affirmatively we have a solid basis for theory upon which the comparison of the radiations of two stars can proceed. Hence duplicate exposures for different given exposure ratios were made for Capella in numerous cases, and for some other stars. For individual exposures, as might be expected, there is hardly any measurable connection between exposure ratios and recorded intensities. One case will be noted where a shorter exposure gives the greater intensity. But it may be assumed that this anomaly in the long run is as much in one direction as in the opposite.

The examination takes the form of determining γp, the photographic development constant multiplied by Schwarzschild's index, separately, for individual wave-lengths over the whole spectrum. The material here discussed was restricted, perhaps unnecessarily, to very low densities, not exceeding about 0·7 at maximum. In consequence of this systematic under-exposure it has been necessary to adapt the usual formula of Hurter and Driffield, connecting exposure with density, in a certain fashion. In spite of this, or perhaps because of it, I think the conclusion will carry conviction that a method is given by which two photograms of a star taken under different but parallel circumstances transform safely into one another, so that there is no risk of drawing fallacious conclusions of difference of state when photograms of two different stars are compared together. Naturally one must qualify this statement by allowing for such small variations as appear in the Polaris series, but these are not at present of importance.

I would remark that in any visual method of photometry all the disabilities and difficulties mentioned above are present, undiminished, but are obscured and cannot be inquired into, because the eye, which serves as the photo-receiver, keeps no record, and further complicates its report by its own peculiarities, among which need only be mentioned three-colour vision and the Purkinje effect.

The theory of the photographic plate, connecting density of image with exposure, is in a somewhat crude state, due not so much to any difficulty of casting it into an appropriate form which shall take account of all essential factors, as because careful experimenters find its behaviour irregular, making uncertain the assignment of definite values to the elements on which it is based. If these values were constants the theory

would be reasonably well established, though perhaps only in empirical shape. The material discussed in the present paper is concerned only with low densities and it is peculiar in that respect. After this peculiarity is allowed for—as may be done very simply—I can find in the material itself no evidence that the elements are not constant; therefore I am satisfied that whatever difficulties may be found in applying it generally, the theory is sufficiently reliable under the circumstances to which it is applied in the present paper.

Let us measure as usual the density of the image D, by $\log_{10}$ (ratio

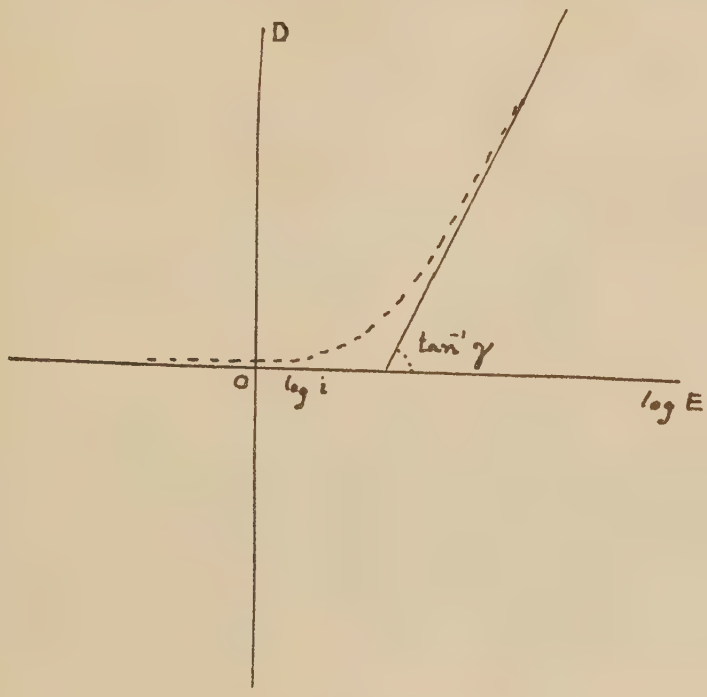


Fig. 3.

of light transmitted through the clear plate to light transmitted through the darkened plate); then if E measures the exposure that produced this density, D is connected with E, according to Hurter and Driffield's theory,* by the equation

$$D = \gamma \log_{10} (E/i)$$

where i, γ are constants, and we call

i = inertia of the plate
γ = development constant.

* *Journ. Soc. Chem. Ind.*, Jan. (1891).

This theory would make the graph of the relation of D and log E a straight line, of pitch γ, and intercept log i on the log E axis. Actually it is known that when the density is low the curve falls away from this line as shown in fig. 3, and it is obvious that ultimately the log E axis must be asymptotic to it. The recognition of this asymptote replaces D on the left by the development $D + a_1/D + a_2/D^2 + \ldots$. To keep the first two terms only treats the graph as an hyperbola. In this form I find the equation satisfies all the requirements of work with low densities, and accordingly write

$$D - a^2/D = \gamma \log_{10} (E/i),$$

introducing thereby a new constant, a^2. Allowing for variation with the wave-length, the equation would run

$$D_\lambda - a_\lambda{}^2/D_\lambda = \gamma_\lambda \log_{10} (E_\lambda/i_\lambda).$$

It will appear below that a_λ shows no marked variation with λ. The subscript λ may therefore be omitted in this case.

Let J_λ measure intensity of emission from the star,
F_λ reducing factor for absorption in the atmosphere,
f_λ reducing factor for absorption in the instrument,
τ exposure time;

then

$$E_\lambda = f_\lambda F_\lambda J_\lambda \tau^p,$$

where p is an index introduced by Schwarzschild,* who records a number of values assigned to it by different experiments, ranging between 0·56 and 0·97. The value $p = 1$, employed before Schwarzschild wrote, gives the "Law of Reciprocity." There seems to be considerable doubt whether p possesses a fixed value over a wide range of exposures,† but we may take it to be a little less than unity. It will be noted that it eliminates itself finally in the applications below, supposing that it does not change appreciably with the wave-length. We have then the equation, connecting emission from the star with density on the photographic plate—

(i) . . . $D_\lambda - a^2/D_\lambda = \gamma_\lambda \log_{10} (f_\lambda F_\lambda J_\lambda \tau^p / i_\lambda)$.

Denote by accented letters corresponding quantities for a second star. f, a, γ, i will be unchanged. Hence

(ii) $(D_\lambda - D_\lambda')(1 + a^2/D_\lambda D_\lambda') = \gamma_\lambda \log_{10} (J_\lambda/J_\lambda') + \gamma_\lambda \log_{10} (F_\lambda/F_\lambda') + p\gamma_\lambda \log (\tau/\tau')$,

the other quantities eliminating themselves. This is the fundamental equation, which may be used in several forms.

[1] Let a series of observations be made of Polaris, with the telescope clamped. Then rigorously $\tau = \tau'$, and except so far as the star's variability

* Schwarzschild (Vienna), *Kuffner. Sternw. Pub.*, Bd. v. (1900), p. 32.

† Shephard and Mees *Theory of the Photographic Process* (1907), p. 220; Kron, *Potsdam Pub. Astrophys. Obs.*, No. 67, Bd. 22 (5), (1913).

is sensible, $J_\lambda = J_\lambda'$. Further, in place of the second star, take the mean of all the exposures, $|D_\lambda|$, $|F_\lambda|$. Then the equation runs

$$(iii) \quad . \quad . \quad (D_\lambda - |D_\lambda|)(1 + a^2/D_\lambda|D_\lambda|) = \gamma_\lambda \log_{10}(F_\lambda/|F_\lambda|),$$

permitting an examination of the effect of atmospheric changes, whether within a few minutes, or from one night to another.

[2] Let pairs of observations of Capella or any suitable star be made, with definite numbers of trails, under circumstances as closely as possible the same. Then $J_\lambda = J_\lambda'$, while τ/τ' is known, and if the series is extensive enough it may be taken that the mean value of $\log(F_\lambda/F_\lambda')$ will be zero. The equation will read

$$(iv) \quad . \quad . \quad |(D_\lambda - D_\lambda')(1 + a^2/D_\lambda D_\lambda')| = p\gamma_\lambda |\log_{10}(\tau/\tau')|,$$

permitting evaluation of $p\gamma_\lambda$ for each wave-length.

[3] Under the same circumstances, in comparing a single pair, while $\log(F_\lambda/F_\lambda')$ may depart considerably from zero, it will remain constant in respect to λ, except in so far as the atmosphere becomes redder or bluer in the interval between the observations, but it will appear from the test [1] above that this is little, when the observations immediately follow one another. Hence forming

$$(v) \quad . \quad (D_\lambda - D_\lambda')(1 + a^2/D_\lambda D_\lambda') = \log_{10}(F_\lambda/F_\lambda') + p\gamma_\lambda \log_{10}(\tau/\tau')$$

for each wave-length, the right-hand side will be nearly constant, except so far as change in γ_λ affects it, and an examination will show the adequacy of the curvature correction in a^2.

[4] It will be seen that these scrutinies of photograms of the same star, taken under different exposures, are a verification that measures taken under the formula employed will give null results where there are no real differences of spectra; in other words, in comparing any two spectra by these means the circumstances of observation are satisfactorily eliminated, and at the same time it is indicated at what point accidental errors qualify this expectation.

[5] Finally, comparing the photograms of any two stars, under atmospheric circumstances assumed to be unvaried, we have the equation

$$(vi) \quad n_\lambda = \gamma_\lambda^{-1}(D_\lambda - D_\lambda')(1 + a^2/D_\lambda D_\lambda') = \log_{10}(J_\lambda/J_\lambda') + p\log_{10}(\tau/\tau'),$$

where τ/τ' must be treated as containing a large accidental unknown part owing to actual, disregarded changes in the sky. This right-hand member might also be written—

$$= (m_\lambda' - m_\lambda) + p\log_{10}(\tau/\tau')$$

introducing a quantity m_λ, which defines relative magnitude for each wave-length. This equation may be considered the answer to Gill's demand, referred to on p. 178, but I shall not proceed further with that idea.

We note that τ (or τ') is absolutely the same for the records of different wave-lengths for the same star, so that if p be taken as a constant, p, τ, τ' may be eliminated from the equation by comparing the

radiations at λ_1, λ_2, and we have the final form of equation, free from all irrelevant circumstances,

$$\text{(vii)} \quad \log_{10}(J_{\lambda_1}/J_{\lambda_1}') - \log_{10}(J_{\lambda_2}/J_{\lambda_2}')$$
$$= \{(D_{\lambda_1} - D_{\lambda_1}')(1 + a^2/D_{\lambda_1}D_{\lambda_1}')\gamma_{\lambda_1}^{-1} - (D_{\lambda_2} - D_{\lambda_2}')(1 + a^2/D_{\lambda_2}D_{\lambda_2}')\gamma_{\lambda_2}^{-1}\}$$
$$= n_1 - n_2, \text{ say,}$$

in which the right-hand member is a measure derived from the photograms, and the left conveys such information as they can supply regarding stellar emission.

This equation will usually be written more conveniently in differential form, taking λ_1, λ_2 adjacent to one another,

$$\text{(viii)} \quad \frac{1}{J}\frac{dJ}{d\lambda} - \frac{1}{J'}\frac{dJ'}{d\lambda} = \frac{1}{\mu}\frac{dn}{d\lambda}$$

where μ is the modulus of common logarithms.

Equations (vii) and (viii), which are now in final form, deserve a remark. They offer measures of stellar radiation as nearly absolute as it is permissible to get. Every irrelevant feature is now eliminated; changing sensitivity of the photographic plate, circumstances of development and exposure, and selective absorption in the telescope and atmosphere, are removed both theoretically and practically, except, one should say, so far as there is a selective difference of atmospheric absorption between the two exposures, or patchy differences in the plate. We proceed to consider and describe the actual measures.

The measures of density D are taken from the photogram by calibrating the photometer by means of a prepared plate of standard densities, the values of which are determined by accepting the result which has been fully verified for the cells in use, that the current in the photoelectric cell is proportional to incident light, and measuring this current with a galvanometer for different grades of density of the plate.

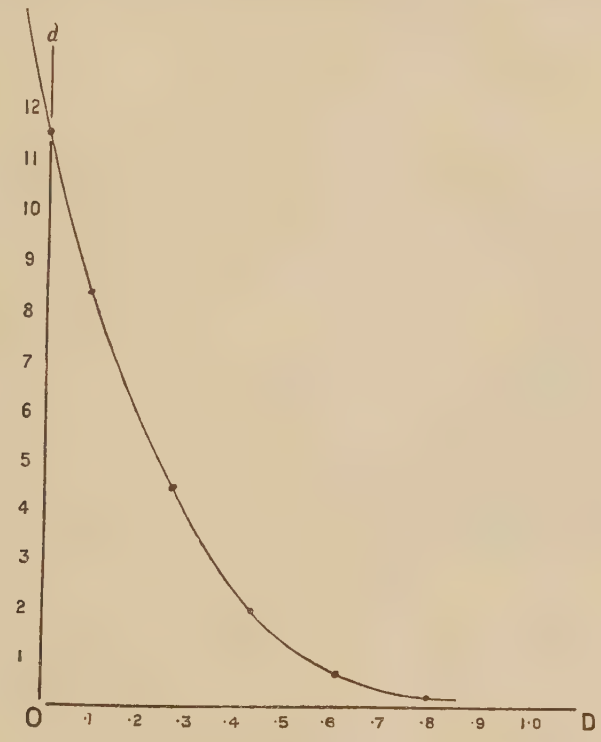
A conversion curve, relating density with the deviation of the thread is obtained. The curve applying to the measures of plates taken since 1922 October 11 is shown in fig. 4.

The readings of density are made by measuring in millimetres the deviation of the image of the thread, as recorded on the photogram, from a datum line registered at the top, which corresponds to infinite density. The Density-Deviation graph of fig. 4 is then available for conversion. It will be seen that this graph marks 11·6 mm. as the deviation corresponding to zero density, that is, to the clear plate. This should correspond to the trace at both ends of the photogram where stellar light has ceased to affect it. It will happen that there are slight differences, even between the readings at the red end of the photogram and the blue end, due no doubt to some patchy variation of clearness of the plate. In practice these differences do not amount to much in the short spectra recorded on our negatives, and are easily treated as "runs" in the following manner:

If AO is the normal deviation from "clear plate" to "full density" [11·6 mm.] but in any special plate there is a defect of amount AN, then in that plate the reading is NO, and the density graph must be taken to start at A′ in place of A, so that for any other measure OM, we read

a density, not MP, but MP – NA′ (fig. 5). This index error NA′ is noted at the beginning and end of each photogram, and applied proportionately for intermediate readings. It is usually zero, mounting rarely to 0·02.

It will be seen that the readings of density become uncertain about, say, 0·65 owing to the steepness of the curve. At the lower end of the scale, on the other hand, they may be vitiated by their own too great sensitiveness, which will show up variations of any sort in the clearness



D, density. *d*, deviation.

Fig. 4.

of the film. In practice it has been found that measures were of normal and apparently of equal value between densities 0·05 and 0·65, and exposures upon stars were made of duration suitable to correspond to this range. The result has the advantage of simplicity, and keeps the whole photogram on the same scale. But restriction of scale to a small range of density is a disability, since it does not permit time enough for the red light of an A star or the blue light of an M star to record itself. It will be avoided in a continuation of the work by breaking the scale, by inserting a suitable obscuring screen in front of the plate when the weak portion of the graph is reached. This device will allow an increased range of density to be covered, step by step.

I now come to the interpretation of the equation (vii) or (viii) as regards the radiations proper to the stars. I have already referred to the uncertainty of relating the physical state of the Sun, which has a mean density exceeding that of water, with Capella, which indeed gives the same spectrum, but is 75 times more luminous, and has a density probably not exceeding ·0002. I merely mention the matter here. If the interpretation of the spectrum by assuming Planck's law to apply, which would assign an equal temperature to both, is conventional, there

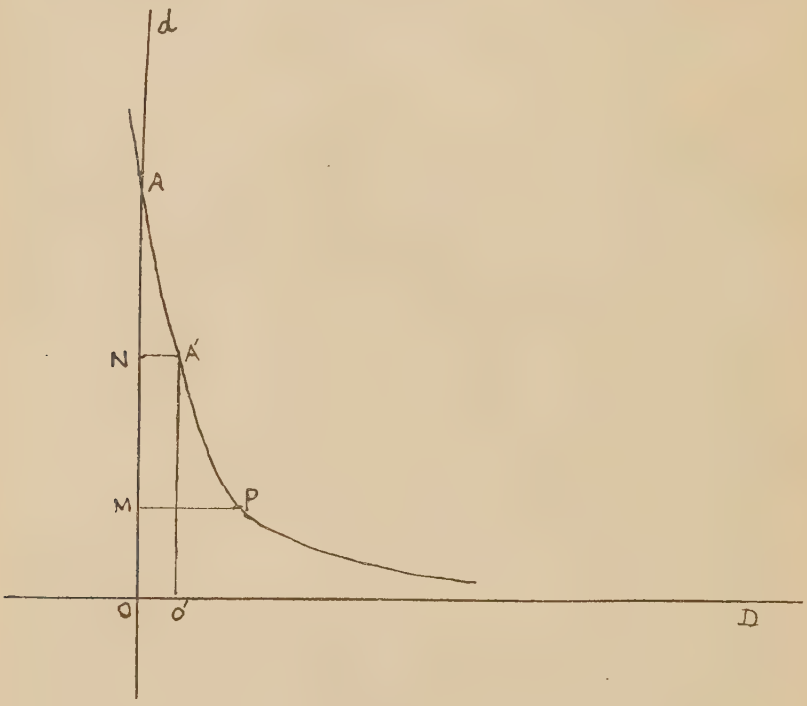


FIG. 5.

is at any rate some real parallelism or identity which such language may be taken as describing. We must, however, recognise the fact that the Sun's radiation differs sensibly from that of a black body, and, besides, if the integrated radiation from the whole disc is taken, a factor is introduced which further alters it systematically. In dealing with giant stars, with their low mean densities, one would suppose that this "atmospheric" modification, which as I have remarked above has the effect of exaggerating the actual temperature difference of two stars, may be very considerable, but we have nothing to go upon. In any case let us write

$$J_\lambda = \psi(\lambda T) I_\lambda = \psi(\lambda T) \cdot \lambda^{-5} / \left(\exp \frac{c_2}{\lambda T} - 1 \right),$$

where I_λ corresponds to black-body radiation at temperature T, and ψ represents any necessary qualifying factor. Equation (viii) then takes the form

$$\text{(ix)} \quad \frac{c_2}{T}N - \frac{c_2}{T'}N' = m - |\chi|_{T'}^{T}$$

where

$$N = 1/\left(1 - \exp\frac{-c_2}{\lambda T}\right),$$

$$m = \frac{\lambda^2}{\mu}\frac{dn}{d\lambda},$$

$$\chi = \frac{\lambda^2}{\psi}\frac{d\psi}{d\lambda}.$$

If λ is expressed in microns the value of C_2 is about 14300. Hence it will be noticed that if the variation of ψ is negligible, and λT does not exceed, say, 4000 or 5000, N closely approaches equality with unity, in which case the final outcome of the observation would be a statement of the difference of the reciprocals of the temperatures of the two stars.

If we could trust that stellar radiation followed Planck's formula exactly, then by comparing two photograms over a range of λ, or even for any two values of λ, and examining the associated changes in the factor N, we could ascertain from a single pair of photograms the temperature of each star, in place of a mere difference. But this would require a degree of precision at present out of reach of observation.

Equation (viii) admits of graphical treatment. Let J as a function of λT be supposed known in form, and graph the function of

$$Z = d(\log J)/d(\log \lambda T)$$

$$= \frac{\lambda}{J}\frac{dJ}{d\lambda}.$$

Then equation (viii) may be written

$$\text{(x)} \quad Z - Z' = \frac{\lambda}{\mu}\frac{dn}{d\lambda}.$$

The right hand is known from the observations. Adopting a value of T, with the assigned value of λ, the abscissa λT and the ordinate Z of the graph are found, and thence the point giving ordinate Z', which in turn gives T'. This applies to (x) treated as an equation giving mere differences. At this stage it is unnecessary to develop the method more generally. We now come to the actual measures of the photograms. The aim is to reach the radiations from some definite stratum of the star, which may be described afterwards. Examining the trace of a star of class B, it will be seen that it shows a general outline which is serrated but which can be smoothed in an easy natural way by running a line through the summits of the teeth, treating the depressions as incipient general absorption. Carrying this practice on to class A,

the continuous line must be run smoothly over the great gaps caused by hydrogen absorption. This is not difficult to do, nor, with these as guide, is it difficult to run the continuous line through the summits for classes F and G; though in the latter case this implies ascription of considerable regions to general absorption. For instance, about λ4227 the continuous line adopted runs well above the trace nearly on the line between the summits about λ4100, λ4360.

It is a matter of judgment, and the agreement with one another of the measures of the Capella series shown below is a fair test of it. With the question, whether this general absorption occurs in the star, or is atmospheric in origin, like the depression shown in all the photograms about the α-band at λ6250, the measures are not concerned. There may be extensive regions of atmospheric general absorption of which we have no record—for example, such a region in the green would account for the absence of green stars. If the absorption is atmospheric, it afterwards eliminates itself by the method of comparing two stars with one another.

Proceeding to the K stars, the previous practice is found to carry on without serious uncertainty as to the location of the continuous line, while for the M stars it is chiefly a question of running it over the great gaps caused by the titanium-oxide bands.

This method of running a continuous line round the photogram has been adopted as an obvious natural beginning of measurement. There is no preliminary evidence that the line belongs to a stratum of uniform temperature, or if it did, as to what that stratum is, in relation to the star's disc. The retention or rejection of the method must depend upon the results it furnishes. In the present paper I aim at exposing these results in outline, as a contribution to progress.

The following is the list of plates taken and stars measured :—

	Star.	Spec., Mag.	[1922·0]. α	δ	Plates.
		m	h m	°	
1	α And	A0p, 2·2	0 4,	+28·7	72
2	β Cas	F5, 2·4	0 5,	58·7	76
3	α Cas	K0, 2·2–2·8	0 36,	56·1	76
4	γ Cas	B0p, 2·3	0 52,	60·3	72, 73
5	β And	M0, 2·4	1 5,	35·2	76
6	α UMi	F8, 2·1	1 33,	88·9	all
7	α Per	F5, 1·9	3 19,	49·6	78
8	α Aur	G0, 0·2	5 11,	45·9	56, 58, 64, 65, 68, 69, 77
9	α Lyr	A0, 0·1	18 34,	38·7	66
10	α Aql	A5, 0·9	19 47,	8·7	46, 54
11	γ Cyg	F8p, 2·3	20 19,	40·0	50, 52, 63
12	α Cyg	A2p, 1·3	20 39,	45·0	51, 54, 55, 62
13	ε Cyg	K0, 2·6	20 43,	33·7	52, 55, 63
14	α Cep	A5, 2·6	21 17,	62·3	67, 70
15	ε Peg	K0, 2·5	21 40,	9·5	75
16	β Peg	M3, 2·2–2·7	23 0,	27·7	71
17	α Peg	A0, 2·6	23 1,	14·8	71, 75

I proceed first to consider the Polaris series of photograms—there are 32 of these. The first 6 are separated sharply from the remainder, by a change in the batch of plates. The new series proved to be much less sensitive in the red than the old. For the purposes of comparison the schedules below are confined to the remaining 26, of which 20 consist of pairs, taken on the same plate within a short time of one another.

Table I. shows: column 1 : spectral positions of measure ; and for the star, column 2 : density of photogram, with the mean at the foot and Julian date of observation. For each wave-length the mean of all the measures was taken and is shown at the beginning of the series. The general mean of this column is also shown at its foot. Column 3 : shows the excess of each measure for any wave-length over the corresponding mean measure, the latter being factored by a suitable multiplier to raise it to a comparable value. The multiplier is the ratio of the mean density for that star to the general mean.

It will be seen that the departure from mean in all cases is small. Their general average, regardless of sign is less than ·00,7. I have not been able to trace any system in them. The Julian date is added in order to see whether they follow the period of variation in magnitude of Polaris. Analysing them it appears they have been taken very near the epochs, 0^d [eleven observations], 1^d [ten observations], 2^d [five observations], 3^d [no observations], counted from the maximum $-2418985^d{\cdot}9 + n \times 3^d{\cdot}9681$. The mean of the excesses is found to be

	0^d.	1^d.	2^d.	3^d.
λ ·36–·42	−·00, 6	−·01, 1	+·02, 6	..
λ ·42–·49	−·00, 1	+·01, 9	−·04, 0	..
λ ·49–·65	−·00, 1	−·02, 1	−·02, 0	..
	[11]	[10]	[5]	[0]

The effect to be expected would be an increase of blueness of the spectrum at maximum.*

There is not much to go upon, but at least we can say that if such an effect were allowed for, it would diminish these residuals.

The range of mean density shows runs from ·08,7 to ·25,0 or from 0·59 to 1·70 of the mean, covering more than one magnitude—a very large variation in general intensity. In contrast with this, any indications of changes in red or blue absorption in the sky from one night to another are inconsiderable. But that they are present, accidentally, in some degree, will be concluded from noting that in pairs, taken from the same plate, there is a general resemblance in the run of the residuals.

Consider now the series of observations of Capella and other stars with varied exposures, made to test the method of judging the identity of any trace. The relevant portion of the theory is [3] and [4] of p. 183, with equation (v)

$$(D_\lambda - D_\lambda')(1 + a^2/D_\lambda D_\lambda') = \log_{10}(F_\lambda/F_\lambda') + p\gamma_\lambda \log_{10}(\tau/\tau').$$

* *Cf.* Shapley. *Ap. J.*, **44** (1916), 273.

TABLE I.—*Polaris Series.*

(1)		Mean Density.	No. 62_2. (2) Measures.	No. 62_2. (3) Excess of Fac. Mean.	No. 62_4. (2) Measures.	No. 62_4. (3) Excess of Fac. Mean.	No. 63_3. (2) Measures.	No. 63_3. (3) Excess of Fac. Mean.	No. 64_2. (2) Measures.	No. 64_2. (3) Excess of Fac. Mean.	No. 65_4. (2) Measures.	No. 65_4. (3) Excess of Fac. Mean.	No. 66_2. (2) Measures.	No. 66_2. (3) Excess of Fac. Mean.	No. 67_2. (2) Measures.	No. 67_2. (3) Excess of Fac. Mean.
H_ζ	3889	·03	·02	1	·02	2	·03	1	·06	*1*	·03	0	·02	0	·06	*2*
	3907	·04	·03	1	·03	2	·05	0	·07	*1*	·05	*1*	·03	0	·08	*2*
Ca	3934 (K)	·06	·05	1	·05	2	·08	0	·10	0	·07	*1*	·04	0	·12	*3*
H_ϵ, Ca	3970 (H)	·09	·07	2	·09	2	·11	1	·14	0	·09	0	·04	1	·15	*2*
Mn	4032	·18	·18	0	·22	0	·24	0	·29	0	·18	0	·08	3	·25	*1*
Sr	+4077	·20	·19	1	·23	1	·28	*1*	·33	*1*	·19	1	·10	2	·29	0
H_δ	4102 (h)	·22	·22	0	·26	0	·29	0	·35	0	·20	1	·12	1	·31	0
	4132	·23	·23	0	·29	*1*	·32	*1*	·37	0	·22	0	·13	1	·33	0
	4173	·25	·25	0	·30	0	·33	0	·39	1	·23	1	·14	1	·36	0
Ca	4227	·26	·28	*2*	·33	*2*	·36	*1*	·41	1	·24	1	·14	2	·38	*1*
	4272	·28	·30	*2*	·37	*3*	·38	0	·43	2	·25	2	·17	0	·40	0
	4308 (G)	·29	·30	*1*	·36	*1*	·39	0	·44	2	·25	3	·17	0	·41	0
H_γ	4341 (g)	·30	·31	*1*	·38	*3*	·41	*1*	·45	3	·25	4	·17	1	·42	1
	4405	·31	·32	*1*	·39	*3*	·43	*2*	·46	4	·28	2	·18	1	·42	2
Mg	+4481	·32	·34	*2*	·42	*4*	·44	*1*	·48	2	·28	3	·19	1	·42	4
Ti	+4550	·32	·32	0	·41	*3*	·45	*2*	·48	2	·29	2	·19	1	·43	3
	4703	·25	·26	*1*	·33	*3*	·36	*3*	·41	*1*	·24	0	·14	1	·35	1
H_β	4861 (F)	·10	·11	*1*	·12	0	·15	*2*	·20	*4*	·13	3	·06	0	·14	0
	5007	·03	·03	0	·02	2	·04	0	·05	0	·05	*2*	·02	0	·05	*1*
Mg	5168 (b)	·02	·02	0	·01	1	·02	1	·04	*1*	·04	*2*	·02	*1*	·04	*1*
	5270 (E)	·03	·02	1	·02	1	·03	1	·06	*1*	·04	*1*	·03	0	·04	0
	5400	·03	·03	0	·02	2	·04	0	·07	*2*	·05	*2*	·04	*2*	·04	0
	5710	·03	·02	1	·02	1	·03	1	·06	*1*	·04	*1*	·03	*1*	·04	0
Na	5893 (D)	·04	·03	1	·02	3	·04	1	·08	*2*	·05	*1*	·04	*1*	·03	3
	6065	·05	·05	0	·05	1	·06	0	·10	*2*	·07	*2*	·04	*1*	·08	*1*
	6253	·04	·04	0	·04	1	·05	0	·09	*3*	·06	*2*	·04	*2*	·05	1
Ca	6450	·07	·07	0	·07	1	·08	1	·14	*4*	·09	*2*	·05	*1*	·09	1
H_α	6563 (C)	·04	·03	1	·03	*2*	·04	1	·09	*3*	·05	*1*	·04	*2*	·05	1
Mean		·14, 7	·14, 7		·17, 5		·19, 7		·23, 7		·14, 3		·08, 8		·20, 8	
1922		..	Oct. 25		Oct. 25		Oct. 25		Oct. 25		Oct. 25		Oct. 25		Oct. 25	

		No. 67_5.		No. 68_1.		No. 68_4.		No. 69_1.		No. 69_3.		No. 70_1.		No. 70_4.		No. 71_2.	
		(2) Measures.	(3) Excess of Fac. Mean.	(2) Measures.	(3) Excess of Fac. Mean.	(2) Measures.	(3) Excess of Fac. Mean.	(2) Measures.	(3) Excess of Fac. Mean.	(2) Measures.	(3) Excess of Fac. Mean.	(2) Measures.	(3) Excess of Fac. Mean.	(2) Measures.	(3) Excess of Fac. Mean.	(2) Measures.	(3) Excess of Fac. Mean.
H_ζ	3889	·06	*1*	·03	0	·06	*2*	·02	1	·02	1	·03	*1*	·03	0	·03	*1*
	3907	·08	*1*	·05	*1*	·07	*2*	·02	2	·03	1	·03	0	·04	0	·03	0
Ca	3934 (K)	·12	*2*	·07	0	·08	0	·06	0	·06	0	·05	*1*	·07	0	·05	0
H_ϵ, Ca	3970 (H)	·16	*1*	·08	2	·13	*2*	·09	0	·10	*1*	·06	0	·12	*2*	·07	0
Mn	4032	·31	0	·20	0	·23	0	·18	*1*	·18	*1*	·11	0	·19	1	·13	1
Sr +	4077	·33	1	·22	0	·27	*2*	·19	0	·20	*1*	·13	1	·23	*1*	·15	1
H_δ	4102 (h)	·36	1	·23	1	·28	*1*	·21	0	·22	*1*	·15	0	·24	1	·17	1
	4132	·37	2	·25	1	·29	0	·22	0	·23	*1*	·15	1	·25	1	·18	0
	4173	·41	0	·28	0	·31	0	·24	0	·25	*1*	·18	0	·29	*1*	·19	1
Ca	4227	·44	0	·29	0	·32	0	·26	*1*	·27	*2*	·19	*1*	·30	*1*	·21	0
	4272	·48	0	·31	0	·34	1	·28	*1*	·28	*1*	·20	0	·32	0	·23	0
	4308 (G)	·50	*1*	·32	0	·35	1	·28	0	·28	0	·20	1	·32	1	·23	0
H_γ	4341 (g)	·50	1	·32	1	·35	1	·30	*1*	·30	*1*	·20	1	·33	1	·23	1
	4405	·53	0	·33	1	·37	2	·30	0	·31	*1*	·24	*2*	·35	0	·25	0
Mg +	4481	·55	*1*	·36	0	·39	1	·32	*1*	·33	*2*	·24	*2*	·36	0	·25	1
Ti +	4550	·55	*1*	·36	0	·39	1	·32	*1*	·33	*2*	·24	*2*	·36	0	·26	0
	4703	·44	0	..		·31	0	·25	*1*	·25	*1*	·18	2	·29	*1*	·18	2
H_β	4861 (F)	·17	0	·12	*1*	·12	0	·09	0	·08	1	·08	*1*	·12	*1*	·07	1
	5007	·06	*1*	·04	*1*	·04	0	·03	0	·02	1	·03	*1*	·05	*2*	·03	*1*
Mg	5168 (b)	·05	*2*	·02	0	·03	0	·02	0	·02	0	·02	0	·02	0	·03	*1*
	5270 (E)	·05	0	·02	1	·03	1	·02	1	·02	1	·02	0	·03	0	·03	*1*
	5400	·06	*1*	·03	0	·04	0	·03	0	·02	1	·03	*1*	·03	0	·03	0
	5710	·05	0	·02	1	·02	2	·02	1	·02	1	·02	0	·03	0	·03	*1*
Na	5893 (D)	·06	1	·04	0	·04	1	·04	0	·03	1	·03	0	·03	1	·03	0
	6065	·08	1	·06	0	·05	1	·05	0	·04	1	·03	1	·05	1	·04	0
	6253	·06	1	·05	*1*	·04	1	·04	0	·03	1	·03	0	·04	1	·03	0
Ca	6450	·11	1	·07	1	·07	2	·07	0	·06	1	·04	1	·07	1	·05	1
H_α	6563 (C)	·07	0	·03	1	·04	1	·04	0	·03	1	·03	0	·04	0	·03	0
	Mean	·25, 0		·16, 3		·18, 1		·14, 2		·14, 3		·10, 5		·16, 4		·11, 9	
	1922	Nov. 1		Nov. 1		Nov. 1		Nov. 1		Nov. 1		Nov. 8		Nov. 8		Nov. 8	
J.D.	2423000^d +	360·3		360·3		360·3		360·3		360·3		367·3		367·3		367·3	

TABLE I.—*continued.*

		No. 71₅.		No. 72₁.		No. 72₄.		No. 73₁.		No. 75₁.		No. 75₄.		No. 76₁.		No. 77₁.	
		(2) Measures.	(3) Excess of Fac. Mean.	(2) Measures.	(3) Excess of Fac. Mean.	(2) Measures.	(3) Excess of Fac. Mean.	(2) Measures.	(3) Excess of Fac. Mean.	(2) Measures.	(3) Excess of Fac. Mean.	(2) Measures.	(3) Excess of Fac. Mean.	(2) Measures.	(3) Excess of Fac. Mean.	(2) Measures.	(3) Excess of Fac. Mean.
H_ζ	3889	·03	*1*	·02	1	·03	*1*	·04	0	·03	0	·03	1	·03	*1*	·02	0
	3907	·03	0	·03	0	·04	*1*	·05	0	·04	0	·05	0	·05	*2*	·03	0
Ca	3934 (K)	·05	0	·04	1	·05	0	·07	0	·06	0	·08	0	·06	*2*	·05	*1*
H_ϵ, Ca	3970 (H)	·06	1	·09	*1*	·08	0	·10	2	·09	0	·10	1	·08	*1*	·08	*2*
Mn	4032	·13	1	·15	0	·15	0	·19	2	·16	1	·22	1	·12	1	·13	0
Sr	+4077	·15	0	·17	0	·17	0	·24	*1*	·19	0	·25	0	·14	1	·14	0
H_δ	4102 (h)	·17	0	·19	0	·18	0	·25	1	·19	2	·27	1	·17	0	·16	*1*
	4132	·18	*1*	·20	0	·19	0	·26	1	·22	0	·28	1	·18	*1*	·17	*1*
	4173	·19	0	·20	1	·21	0	·29	0	·23	1	·30	*1*	·19	0	·18	0
Ca	4227	·20	*1*	·21	1	·23	*2*	·30	0	·24	1	·33	0	·19	0	·19	*1*
	4272	·21	0	·23	1	·24	*1*	·32	1	·26	1	·35	0	·21	0	·21	*1*
	4308 (G)	·22	0	·24	1	·24	0	·33	1	·27	1	·36	0	·21	1	·21	*1*
H_γ	4341 (g)	·22	0	·25	1	·25	0	·34	1	·28	1	·37	1	·22	1	·22	*1*
	4405	·24	*1*	·25	2	·26	0	·34	2	·29	1	·37	2	·23	0	·22	0
Mg	+4481	·25	*1*	·26	1	·27	0	·37	0	·30	1	·40	0	·23	1	·23	*2*
Ti	+4550	·25	*1*	·26	1	·27	0	·37	0	·30	1	·41	*1*	·24	0	·23	*2*
	4703	·19	2	·20	1	·19	2	·29	0	·24	0	·31	0	·18	1	·17	2
H_β	4861 (F)	·06	*2*	·08	1	·08	0	·12	0	·10	0	·13	0	·07	0	·05	2
	5007	·02	0	·03	*1*	·03	0	·04	0	·04	*1*	·05	*1*	·02	0	·02	0
Mg	5168 (b)	·02	0	·03	*2*	·02	0	·03	0	·04	*2*	·04	*1*	·02	0	·01	0
	5270 (E)	·02	0	·03	0	·02	0	·04	0	·04	*1*	·04	0	·02	0	·01	1
	5400	·03	*1*	·04	*2*	·03	*1*	·04	0	·04	*1*	·05	*1*	·02	0	·02	0
	5710	·03	*1*	·03	0	·02	0	·03	1	·04	*1*	·06	*1*	·03	*1*	·02	0
Na	5893 (D)	·03	0	·04	*1*	·03	0	·05	0	·05	*1*	·07	*2*	·04	*1*	·02	1
	6065	·04	0	·07	*2*	·04	0	·08	*2*	·07	*2*	·09	*3*	·05	*1*	·02	2
	6253	·03	0	·05	*2*	·04	*1*	·07	*2*	·05	*1*	·08	*3*	·04	*1*	·02	1
Ca	6450	·05	0	·09	*3*	·06	0	·12	*4*	·09	*2*	·12	*3*	·07	*2*	·03	2
H_α	6563 (C)	·03	0	·05	*2*	·04	0	·06	*1*	·06	*2*	·07	*2*	·04	*1*	·02	1
	Mean	·11, 2		·12, 6		·12, 4		·17, 2		·14, 3		·18, 8		·11, 2		·10, 3	
	1922	Nov. 8		Nov. 8		Nov. 8		Nov. 13		Nov. 17		Nov. 17		Nov. 17		Nov. 24	
J.D.	2423000[d]+	367·3		367·3		367·3		372·3		376·3		376·3		376·3		383·3	

TABLE I.—*continued.*

		No. 77$_4$.		No. 78$_1$.		No. 78$_4$.	
		(2) Measures.	(3) Excess of Fac. Mean.	(2) Measures.	(3) Excess of Fac. Mean.	(2) Measures.	(3) Excess of Fac. Mean.
H_ζ	3889	·02	0	·02	0	·02	0
	3907	·02	1	·02	1	·02	0
Ca	3934 (K)	·04	0	·04	0	·04	0
H_ϵ, Ca	3970 (H)	·07	*1*	·06	0	·07	*2*
Mn	4032	·13	0	·12	0	·11	0
Sr+	4077	·16	*2*	·14	*1*	·12	0
H_δ	4102 (L)	·17	*2*	·16	*1*	·13	0
	4132	·18	*2*	·16	*1*	·14	0
	4173	·18	0	·18	*1*	·15	0
Ca	4227	·20	*2*	·18	*1*	·15	0
	4272	·22	0	·20	*1*	·17	0
	4308 (G)	·22	0	·20	0	·18	*1*
H_γ	4341 (g)	·23	*2*	·21	*1*	·18	*1*
	4405	·23	*1*	·21	0	·18	1
Mg+	4481	·24	*2*	·20	1	·19	0
Ti +	4550	·24	*2*	·23	*2*	·19	0
	4703	·18	0	·17	0	·14	1
H_β	4861 (F)	·05	2	·05	2	·06	0
	5007	·01	1	·02	0	·02	0
Mg	5168 (b)	·00	1	·01	0	·02	*1*
	5270 (E)	·00	2	·01	1	·02	0
	5400	·01	1	·02	0	·02	0
	5710	·01	1	·02	0	·01	1
Na	5893 (D)	·02	1	·02	1	·02	0
	6065	·02	2	·03	1	·02	1
	6253	·01	2	·03	0	·02	0
Ca	6450	·02	3	·04	1	·03	1
H_α	6563 (C)	·01	2	·03	0	·02	0
	Mean	·10,	3	·09,	9	·08,	7
	1922	Nov. 24		Nov. 24		Nov. 24	
J.D.	2423000^d+	383·3		383·3		383·3	

By trial it was found that the value

$$a^2 = \cdot 0150$$

brought low densities into satisfactory accordance with higher densities. This is readily tested at the places where there is a steep rise or fall of density within a short range of wave-length. The construction of Table II. is then as follows:—Taking the first observation No. 46, Polaris, with exposure ratio $\tau/\tau' = 1 \cdot 5$, the two exposures gave for $\lambda 3934$, $D = \cdot 13$; $D' = \cdot 06$ (column 2), from which the left-hand member is found

$$\cdot 07 \times 2 \cdot 9 = \cdot 20 \text{ (column 3)},$$

and similarly for other cases. The entries are given to unit ·01, decimal points being omitted. All observations are included in which the lower density measured does not fall below ·04. At the foot of column 3 stands $\cdot 176 = \log_{10}(\tau/\tau')$. The run of numbers in Column 3 is not smooth for a single observation, nor can this be expected. The presence of the atmospheric term log (F/F′) makes it impossible to draw any conclusion from a single observation. But if collected together in a sufficiently numerous series the mean value of log (F/F′) should be zero, and the numbers would then supply a determination of $p\gamma_\lambda$ or p being supposed given, the development constant γ_λ for any wave-length. When the whole series is collected without discrimination, results are found as shown in the collection. They divide into groups which are fairly consistent with themselves. A number of low values, about 0·72, appear, which on examination prove to be due to the non-measurement of certain A stars at these points. It is also seen that the values beyond $\lambda 4861$ depend upon very few records. As far as its evidence goes it indicates not much change of the quantity γ_λ with the wave-length. But it is clear that the series is not extensive enough to serve as a reliable determination of it. As the numerical value of γ_λ is essential for converting other measures, I made inquiry of Mr. B. V. Storr, Chief Chemist of Ilford Ltd., who kindly gave me the following information:—In experiments conducted in the Research Laboratory of the Company, γ was found to vary with the wave-length: "γ is lower for each end of the spectrum than it is for the yellow and yellow-green region. In some experiments made here in the early part of 1919, this point was tested; the results were not perfectly regular, but the general statement given above as to the variation of γ with the wave-length was fully borne out, and a total range from about 1·45 up to about 1·9 was obtained." Considering this information, and in view of the incompleteness of the stellar material, it seemed better to disregard the value indicated by the collection of Table II. and employ a figure midway in the range given by Mr. Storr. The mean value 1·7 was adopted, which also lies within the range of variation of individual stellar observations in Table II.*

A variation of γ with wave-length would introduce no difficulty, *cf.* equations (vi), (viii), pp. 183, 184. But at the present stage it is not allowed for.

* Mr. Storr expresses the opinion (1923 March) that this value is too large. But I let it stand, pending a direct determination.

TABLE II.

Exposure Differences.

	(1)	(46) α UMi. τ/τ′=1·5. (2)	(3)	(55) α Cyg. τ/τ′=16/11. (2)	(3)	(56) α Aur. τ/τ′=7/5. (2)	(3)	(56) α Aur. τ/τ′=7/4. (2)	(3)	(62) α Cyg. τ/τ′=14/13. (2)	(3)	(64) α Aur. τ/τ′=6/4. (2)	(3)	(64) α Aur. τ/τ′=7/4. (2)	(3)	(64) α Aur. τ/τ′=7/5. (2)	(3)
Ca	3934 (K)	13, 6	=20	37, 23	=16	6, 4	=14	6, 3	=..	23, 25	=−3	4, 3	=..	..	=..	4, 4	=..
H_ϵ, Ca	3970 (H)	18, 8	20	42, 32	11	9, 7	7	9, 6	11	28, 33	−6	6, 3	..	6, 3	..	6, 6	0
Mn	4032	35, 17	22	56, 38	19	22, 16	9	22, 13	17	34, 39	−6	12, 8	10	14, 8	14	14, 12	4
Sr+	4077	36, 21	18	61, 42	20	25, 18	9	25, 16	12	38, 43	−6	13, 10	7	15, 10	10	15, 14	2
H_δ	4102 (h)	40, 22	21	61, 45	17	27, 19	10	27, 17	13	39, 43	−4	16, 12	7	16, 12	7	16, 16	0
	4132	45, 24	24	65, 46	20	28, 21	9	28, 18	13	41, 44	−3	16, 12	7	17, 12	8	17, 16	2
	4173	48, 27	23	68, 46	23	31, 22	11	31, 19	15	41, 45	−4	17, 13	7	18, 13	8	18, 18	0
Ca	4227	51, 30	23	68, 48	21	34, 24	12	34, 22	14	42, 47	−5	19, 14	8	20, 14	9	20, 21	−1
	4272	58, 32	28	..		37, 28	11	37, 25	14	..		21, 15	9	21, 15	7	21, 21	0
	4308 (G)	58, 33	27	63, 48	16	38, 30	9	38, 27	13	43, 45	−2	22, 16	8	22, 16	8	22, 22	0
H_γ	4341 (g)	58, 34	26	63, 48	16	42, 33	10	42, 28	16	41, 45	−4	23, 18	7	23, 18	7	23, 23	0
	4405	58, 35	25	62, 45	18	47, 36	12	47, 32	16	40, 45	−5	24, 20	5	26, 20	8	26, 26	0
Mg+	4481	70, 38	..	58, 42	17	55, 39	17	55, 36	21	38, 43	−6	27, 22	7	29, 22	8	29, 28	1
Ti +	4550	70, 39	..	55, 41	15	55, 41	15	55, 36	21	36, 42	−7	29, 24	6	30, 24	7	30, 29	1
	4703	54, 32	24	45, 30	17	46, 36	11	46, 32	16	28, 31	−4	24, 19	7	23, 19	5	23, 26	−3
H_β	4861 (F)	30, 17	17	22, 13	14	28, 21	9	28, 17	15	8, 11	−8	13, 9	9	14, 9	11	14, 14	0
	5007	20, 8	23	8, 4	23	14, 8	14	14, 7	18	2, 2	..	..		6, 3	..	6, 7	−4
	5270 (E)	20, 7	27	..		13, 8	12	13, 6	20	..		5, 3	..	6, 3	..	6, 6	0
	5400	23, 10	21	6, 3	..	18, 12	10	18, 7	24	..		6, 3	..	7, 3	..	7, 6	5
	6065	41, 21	23	..		31, 19	15	31, 16	19	..		12, 7	14	14, 7	18	14, 13	2
Ca	6450	58, 25	36	..		45, 28	19	45, 23	24	..		17, 11	11	19, 11	14	19, 17	3
H_α	6563 (C)	38, 17	26	..		29, 13	20	29, 13	22	..		12, 7	13	14, 7	18	14, 12	4
log τ/τ′=		·176		·162		·146		·243		·032		·176		·243		·146	

TABLE II.—*continued.*

	(1)	(65) α Aur. τ/τ′=5/4 (2)	(3)	(65) α Aur. τ/τ′=6/4 (2)	(3)	(66) α Lyr. τ/τ′=5/3 (2)	(3)	(66) α Lyr. τ/τ′=6/3 (2)	(3)	(66) α Lyr. τ/τ′=6/4 (2)	(3)	(67) α Cep. τ/τ′=4/3 (2)	(3)	(68) α Aur. τ/τ′=6/4 (2)	(3)	(69) α Aur. τ/τ′=6/4 (2)	(3)
Ca	3934 (K)	2, 2	=..	3, 2	=..	16, 6	=15	35, 6	=49	35, 13	=29	31, 23	=10	2, 1	=..	..	
H_ϵ, Ca	3970 (H)	3, 2	..	5, 2	..	19, 7	25	40, 7	50	40, 16	29	38, 29	11	2, 2	..	6, 3	=..
Mn	4032	7, 5	11	7, 5	11	24, 10	22	50, 10	52	50, 20	34	49, 39	11	6, 3	..	12, 3	..
Sr+	4077	8, 7	4	12, 7	14	..		..		..		..		7, 4	..	14, 5	28
H_δ	4102 (h)	9, 8	3	13, 8	8	26, 13	18	55, 13	50	55, 23	35	53, 43	11	9, 4	25	17, 7	23
	4132	9, 8	3	13, 8	8	..		..		..		..		10, 5	20	17, 7	23
	4173	10, 9	3	15, 9	13	..		..		..		..		12, 6	20	17, 8	19
Ca	4227	12, 10	4	16, 10	11	29, 14	21	58, 14	52	58, 24	38	58, 48	11	13, 7	17	22, 9	23
	4272	13, 12	2	17, 12	8	..		..		..		..		..		23, 9	25
	4308 (G)	14, 12	4	18, 12	10	..		..		..		..		17, 8	19	23, 10	22
H_γ	4341 (g)	15, 14	2	19, 14	8	28, 14	20	58, 14	52	58, 24	38	55, 44	12	18, 9	17	27, 11	22
	44c5	17, 14	5	21, 14	11	..		..		..		..		19, 11	14	30, 13	24
Mg+	4481	19, 17	3	24, 17	10	28, 13	21	58, 13	54	58, 23	39	57, 44	14	25, 13	18	33, 14	24
Ti+	4550	20, 18	3	24, 18	8	26, 13	18	56, 13	52	56, 22	38	55, 43	13	27, 13	20	34, 15	24
	4703	16, 14	4	21, 14	10	20, 8	22	44, 8	50	44, 15	35	44, 33	12	20, 10	18	27, 12	21
H_β	4861 (F)	8, 8	0	12, 8	6	6, 3	..	18, 3	..	18, 5	35	19, 13	10	8, 4	..	12, 3	..
	5007	3, 2	..	4, 3	..	1, 1	..	6, 1	..	6, 1	..	8, 4	..	..		3, 1	..
	5270 (E)	2, 2	..	3, 2	..	..		..		..		..		..		..	
	5400	3, 2	..	4, 2	..	..		..		..		..		..		..	
	6065	5, 4	..	9, 4	..	..		..		..		..		..		..	
Ca	6450	8, 6	8	13, 6	13	..		..		..		..		..		..	
H_α	6563 (C)	5, 4	..	8, 4	..	..		..		..		..		..		..	
log τ/τ′=		·097		·176		·222		·301		·176		·145		·176		·176	

TABLE II.—*continued.*

	(70) α Cep. $\tau/\tau'=9/6$. (2)	(3)	(77) α Aur. $\tau/\tau'=7/5$. (2)	(3)	(78) α Per. $\tau/\tau'=8/6$. (2)	(3)	*Collection.* Sum of Col. (3).	Coeff.	$\gamma_\lambda p$.
Ca 3934 (K)	14, 7	=18	..	..	14, 8	=14	1·82	1·82	1·00
H_e, Ca 3970 (H)	18, 10	14	8, 2	=..	18, 12	10	1·82	2·06	0·88
Mn 4032	..		13, 5	18	27, 18	12	2·60	2·74	0·95
Sr+4077	..		15, 7	19	30, 20	15	1·52	2·05	0·74
H_δ 4102 (h)	29, 17	16	18, 9	17	31, 21	14	2·91	3·25	0·90
4132	..		19, 9	19	32, 22	14	1·67	2·23	0·75
4173	..		20, 10	18	35, 24	13	1·69	2·23	0·76
Ca 4227	32, 19	16	22, 12	16	37, 25	14	3·04	3·25	0·95
4272	..		24, 13	16	37, 27	12	1·32	1·86	0·71
4308 (G)	..		24, 14	15	37, 27	12	1·60	2·23	0·72
H_γ 4341 (g)	32, 19	16	25, 14	16	37, 27	12	2·93	3·25	0·90
4405	..		29, 17	16	38, 27	13	1·62	2·23	0·73
Mg+4481	31, 18	17	31, 18	17	39, 28	12	2·94	3·07	0·96
Ti +4550	29, 16	17	33, 19	17	39, 27	13	2·81	3·07	0·92
4703	23, 14	13	24, 13	16	31, 18	17	2·91	3·25	0·89
H_β 4861 (F)	7, 4	18	9, 4	..	10, 5	20	2·07	2·63	0·79
5007	..		..		3, 2	..	·74	·73	0·73
5270 (E)	..		..		2, 2	..	·59	·57	1·04
5400	..		..		2, 2	..	·55	·57	0·96
6065	..		..		4, 3	..	·91	1·13	0·81
Ca 6450	..		..		6 3	..	1·28	1·40	0·91
H_α 6563 (C)	..		..		3, 2	..	1·03	1·13	0·91
$\log \tau/\tau' =$	·176		·146		·135				

We come next to the comparisons of photograms of two different stars, in order to deduce information as to their relative temperatures, by means of the formula of equation (ix), p. 187.

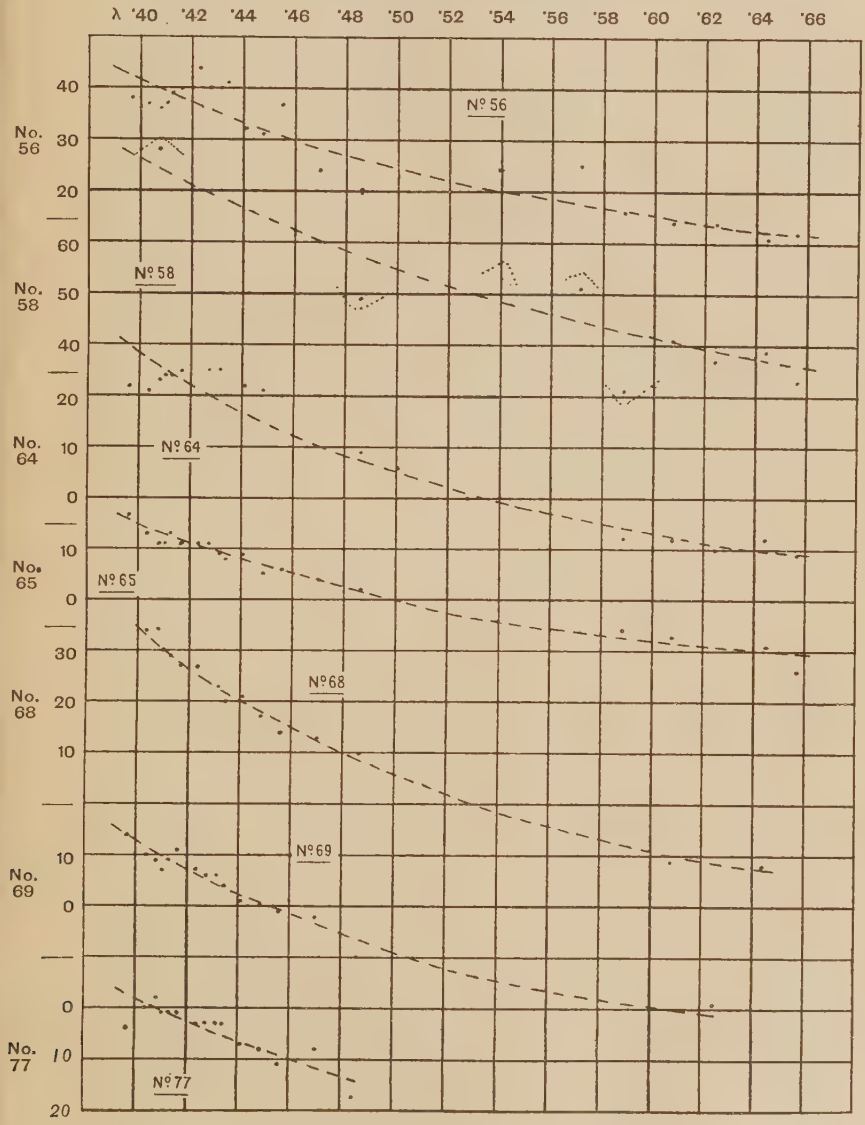
Only photograms of spectra taken on the same plate within a short time of one another are brought into comparison. As a regular standard of reference, Polaris is used. Polaris itself was standarised by seven comparisons with Capella, as it was intended to enter the work by adopting a temperature for the latter. The values of

$$n\gamma=(D-D')(1+a^2/DD')$$

are then found, for such values of the wave-length as admit reliable measurement for both stars, and are charted upon a graph. Strictly, the proper numbers to chart would be n, recognising thus the variation of γ with wave-length, but γ is here taken at a constant value. A line is drawn by hand representing the estimated smoothed run of the whole of the measures, and $\gamma dn/d\lambda$, the gradient of this line, is measured. When the comparison covers the whole range of λ, the measurement is made separately at $\lambda\lambda$ ·4, ·5, ·6 for use in the subsequent calculations.

After the particulars given in the previous collections, it will be unnecessary to publish here the whole of the individual measures. The attached set, showing the plotted measures and graphs for the set of comparisons of Polaris and Capella may be taken as representative of

the material. It is neither better nor worse than the average, except that for these two stars it is usually possible to obtain readings for both



Series of Polaris-Capella Comparisons.

over the whole range of λ, whereas for A stars there would as a rule be no comparison in the red, and for K and M stars some of the measures in the blue would be impracticable.

The further reduction now runs as follows:—If the readings were strong enough, separate determinations might be made for each wave-length over the whole spectrum, which could be made to furnish in effect a comparison of the measured radiations *for each star*, with those given by Planck's curves for chosen appropriate temperatures; and no doubt differences would appear, as they appear for the Sun, and they might be significant. But this is a refinement not in reach of these first measures. The three measures made for λλ ·4, ·5, ·6 are accordingly united in a mean, to give a single result, multiplying each first by λ^2 in order to put them on a comparable basis, and giving double weight to the measure for λ·5 in the middle of the comparison curve, to which wave-length the outcome is supposed to apply. In Table III. subjoined, for Polaris and each other star, the number of the plate is shown in column 1; the actual values measured for $\gamma dn/d\lambda$ are shown in columns 2, 3, 4. In columns 5, 6, 7 the associated values of $\gamma\lambda^2 dn/d\lambda$ are given. Column 8 is the general mean, with double weight for λ·5, as just explained, and the same quantity multiplied by $1/\gamma\mu$ is shown in column 9, giving $\mu^{-1}dn/d\lambda$, the right-hand member of equation (viii) for use in the subsequent calculation, where γ is the development coefficient, adopted at the constant value 1·7 as explained on p. 194, and $\mu = ·4343$, the modulus of common logarithms.

TABLE III.

Comparative Measures of Stars.

		$-\gamma dn/d\lambda$			$-\gamma\lambda^2 dn/d\lambda$			Mean.	Col. 8 $\div\gamma\mu$
	(1).	(2).	(3).	(4).	(5).	(6).	(7).	(8).	(9).
(1)									
Polaris	(56)	1·85	1·29	0·81	·296	·325	·292		
Capella	(58)	2·15	1·66	1·16	·344	·415	·418		
	(64)	2·20	1·65	0·97	·352	·413	·349		
	(65)	1·70	1·05	0·62	·272	·263	·223		
	(68)	4·08	2·13	1·10	·652	·533	·400		
	(69)	2·97	1·54	0·75	·475	·385	·270		
	(77)	1·97	1·62	..	·315	·405	..	·373	·506
(2)									
Vega	(66)	3·54	2·63	..	·566	·658	..	·627	·851
Polaris									
(3)									
α Cep.	(67)	2·82	1·51	0·91	·451	·378	·328		
Polaris	(70)	2·92	1·92	..	·467	·480	..	·423	·573
(4)									
Polaris	(76)	4·80	2·90	1·76	·768	·725	·634	·713	·968
α Cas.									
(5)									
Polaris	(76)	..	5·62	..	..	1·485	..	1·485	2·015
β And.									

TABLE III.—*continued.*

		$-\gamma dn/d\lambda$			$-\gamma\lambda^3 dn/d\lambda$		Mean.	Col. 8. $\div\gamma\mu$.
	(1).	(2).	(3).	(4).	(5).	(6).	(7).	(8).	(9).
(6)									
α Per.	(78)	1·22	0·50	..	·195	·125	..	·148	·201
Polaris									
(7)									
α Aql.	(46)	2·22	1·80	1·18	·355	·450	·425		
Polaris	(54)	3·60	2·68	..	·576	·670	..	·514	·697
(8)									
Polaris	(50)	0·24	0·18	0·12	·038	·045	·043		
γ Cyg.	(52)	0·56	0·24	0·14	·090	·060	·050		
	(63)	0·26	0·22	0·14	·042	·055	·050	·053	·072
(9)									
	(51)	5·56	2·96	..	·890	·740	..		
α Cyg.	(54)	5·18	2·80	1·18	·829	·700	·425		
Polaris	(55)	4·32	2·52	2·06	·691	·630	·742		
	(62)	4·54	3·16	..	·726	·790	..	·716	·972
(10)									
	(52)	3·22	2·28	1·74	·515	·570	·626		
Polaris	(55)	3·00	1·66	0·92	·480	·415	·331		
ε Cygni	(63)	3·70	2·58	1·76	·592	·645	·634	·537	·729
(11)									
Polaris	(71)	..	5·76	..	..	1·440	..	1·440	1·954
β Peg.									
(12)									
α Peg.	(71)	4·70	3·54	..	·752	·885	..		
Polaris	(75)	5·22	3·16	..	·835	·790	..	·823	1·117
(13)									
β Cas.	(76)	2·28	1·56	0·80	·365	·390	·288	·358	·485
Polaris									
(14)									
α And.	(72)	5·60	3·30	..	·896	·825	..	·849	1·152
Polaris									
(15)									
γ Cas.	(72)	..	3·30	..	..	·825	..		
Polaris	(73)	6·36	3·54	..	1·018	·885	..	·888	1·205
(16)									
Polaris	(75)	..	4·28	2·54	..	1·070	·914	1·018	1·381
ε Peg.									

We must now examine such material as is available for estimating the terms $\psi^{-1}d\psi/d\lambda$ in equation (ix), due to reddening of the star's effective radiation, by integration over the disc. Very complete data are given by Abbot,* which show the progressive reddening of zones of the solar disc compared to the radiation of the same wave-lengths at the centre from $\lambda\cdot323$ to $\lambda2\cdot097$. Denoting the entry of this table as $\phi(r, \lambda)/(\phi(0, \lambda)$ where r is the radius of the ring of the disc referred to, with the full radius as unit, we have for the integrated radiation J_λ,

$$J_\lambda = \frac{1}{\pi a^2}\int_0^a 2\pi r dr \phi(r, \lambda) I_\lambda$$

$$= I_\lambda \int_0^1 2r dr \phi(r, \lambda) \qquad [a = 1]$$

$$= I_\lambda \psi(\lambda),$$

as written on p. 186. Hence by mechanical integration over the solar disc the value of $\psi(\lambda)/\phi(0, \lambda)$ may be derived from Abbot's table. As to the value of $\phi(0, \lambda)$, which is the coefficient of absorption at the centre of the disc, we have no information. If we take it as unity, that assumes that the stratum from which Abbot's standard measures of the Sun are derived is equivalent to the line of summits through which measures of the photograms were made. Let this be assumed provisionally, and we have the integrated reddening of the solar disc. The effect, it may be remarked, is much greater than might be anticipated, owing to the preponderating area of the outer zones where the relative reddening is greatest.

There are two assumptions which it is necessary to make if we are to derive from these numbers corresponding factors to be applied to the stars. They have been mentioned already on pp. 178, 179. The first is, that numbers derived from a dwarf star may be carried over, unchanged, to a giant star of the same spectral type; and the second, that for stars of different temperature the value of ψ is the same, for the same value of λT. It is unnecessary to argue about these assumptions. If any assumptions must be made they are the obvious ones for trial. One can only expose them clearly and examine whether they fit the phenomena.

Mechanical integration of Abbot's data supplies the following results:—

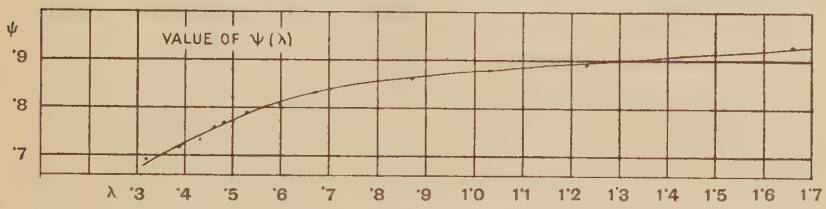
λ.	$\psi(\lambda)$.	λ.	$\psi(\lambda)$.	λ.	$\psi(\lambda)$.	λ.	$\Psi(\lambda)$.
·323	·694	··481	·771	·670	·835	1·225	·887
386	·716	·501	·774	·699	·840	1·655	·925
·433	·732	·534	·791	·866	·863	2·097	·933
·456	·757	·604	·813	1·031	·884		

These are shown on the graph below. The large proportions reached by the general reduction from the integrated disc, relative to values at the

* *Ann. Smith. Astrophys. Obs.*, 3 [1913], 157, Table LV.

centre, will be remarked, and its rapid increase with diminishing wavelengths.

From this graph the values of ψ and $d\psi/d\lambda$ were read for a number of values of λ, and, assuming that the solar data refer to temperature



6000° A., the following table of values of $\psi^{-1}d\psi/d\lambda$ for different wavelengths and for a sequence of temperatures was formed.

TABLE IV.

T=	4000°	5000°	6000°	7000°	8000°	9000°	10,000°	12,000°	15,000°
$\psi^{-1}d\psi/d\lambda$.	λ.	λ.	λ.	λ.	λ.	λ.	λ.	λ.	λ.
·81	·45	·36	·30						
·68	·60	·48	·40	·34	·30				
·59	·75	·60	·50	·43	·38	·33	·30		
·46	..	·72	·60	·52	·45	·40	·36	·30	
·29	..	..	·70	·60	·53	·47	·42	·35	·28
·19	..	..	·80	·69	·60	·53	·48	·40	·32
·10	..	..	·90	..	·68	·60	·54	·45	·36
·10	..	..	1·00	..	..	·67	·60	·50	·40

In particular, for λ·5, to which the actual measures of the photograms are reduced, we have

TABLE V.

	4000°.	5000°.	6000°.	7000°.	8000°.	9000°.	10,000°.	12,000°.	15,000°.
$\psi^{-1}d\psi/d\lambda$	·77	·67	·59	·49	·35	·24	·16	·10	·10
$\lambda^2\psi^{-1}d\psi/d\lambda$	·19	·17	·15	·12	·09	·06	·04	·03	·02

With these numbers we may now proceed to apply equation (ix), p. 187, to determine the relative temperatures of the stars whose measures are given in Table II.

$$\frac{c_2}{T}N - \frac{c_2}{T'}N' = m - \chi_T + \chi_{T'}$$

where

$$N = 1/\left(1 - \exp.\frac{-c_2}{\lambda T}\right), \quad \chi = \lambda^2\psi^{-1}d\psi/d\lambda.$$

The value of c_2 adopted is 14300.
The equation is solved for λ·5.

For this value of λ the factor N has the following values

TABLE VI.

$\lambda \cdot 5$, *Value of* $log_{10}N$.

T.	$log_{10}N$.	T.	$log_{10}N$.	T.	$log_{10}N$.	T.	$log_{10}N$.
4000°	·0003	8,000°	·0123	12,000°	·0421	16,000°	·0796
5000	·0014	9,000	·0185	13,000	·0510	17,000	·0893
6000	·0037	10,000	·0256	14,000	·0603	18,000	·0992
7000	·0074	11,000	·0335	15,000	·0698	19,000	·1089
						20,000	·1188

By adopting a value for T′, the value of T may be readily deduced, from equation (ix), by trial and error, the subsidiary quantities χ, N, changing relatively little in comparison with changes in T. It will be observed, however, that the factor N increases with T, and in fact $Lt_{T\to\infty} c_2 N/T = \lambda$, so that as T increases its exact determination becomes more difficult, in parallel with the difficulty of distinguishing the exact form of the Planck radiation curve as it becomes steeper when λ_{max} diminishes. The temperatures with which we deal do not raise this difficulty in serious form, but the solution shows it by giving for the higher temperatures distinctly higher final values when the change of N is recognised than a first approximation with $N = 1$ would indicate, and correspondingly the equivalent in change of temperature of measured changes of form of the photograms becomes greater and greater as the temperature rises.

The present investigation requires an adopted value for T′. It was thought that with the Sun as parallel a satisfactory value could be adopted for Capella, and that for Polaris being deduced by numerous comparisons of the two stars, the remaining stars could then be referred to Polaris. Examination of the photograms of Capella show it to be very distinctly suppressed in the blue in comparison with Polaris, as though marked by the general absorption in that region which becomes so prominent in the K stars that follow it in spectral order. It will be recalled that the measured solar radiations fall sensibly below Planck's curve in the blue. Whether the spectrum of Capella differs in this respect from the solar spectrum it is impossible to be sure at present. It was a case in which judgment must be exercised, and I judged that a better series of comparative values for temperatures would be found by adopting the temperature for Capella rather below the solar temperature of 6000°, say 5500° A. This number is the basis of the calculations that follow.

Arranged in order of descending temperature, the following results are shown. The spectral classes and absolute magnitudes, when known, are also given, as well as Wilsing's temperatures, for comparison.

TABLE VII.

Resultant Temperatures.—[*Capella* 5500°].

			M.	[Wilsing].
16,900°	γ Cas.	Bop	..	[6,800°]
15,500	α And.	Aop	1·1	[9,400°]
15,000	α Peg.	Ao	..	[10,500°]
12,500°	α Cyg.	A2p	..	[9,400°]
11,200	Vega	Ao	0·6	[9,400°]
10,000	α Aql.	A5	1·7	[8,100°]
9,200°	α Cep.	A5	1·3	[6,900°]
8,800	β Cas.	F5	1·6	[5,200°]
7,500	α Per.	F5	−1·3	[5,700°]
6,800°	Polaris	F8	−3·0	[5,600°]
6,600	γ Cyg.	F8p	−3·0	[6,300°]
[5,500]	Capella	Go	0·1	[7,100°]
5,100°	ϵ Cyg.	Ko	0·8	[4,400°]
4,700	α Cas.	Ko	−1·0	
4,200	ϵ Peg.	Ko	−0·3	[4,500°]
3,600°	β Peg.	M3	0·0	[2,800°]
3,500	β And.	Mo	0·5	[3,200°]

The order of this table follows so closely the order of spectral class, that a few remarks upon it may be made from that point of view.

γ Cas.—This is a star of type Bop, with bright hydrogen lines. Wilsing gives the temperature 6800°. H. H. Plaskett [*M.N.*, **80**, 1920, 771] also found 6800°. The photogram shows a star sensibly bluer than α And. and α Peg. It would be impossible to read a relatively low temperature from it.

The four stars α And. (Aop), α Peg. (Ao) α Cyg. (A2p) and Vega (Ao) may be considered together. The three stars marked Ao show a well-marked progression in the sense of increasing breadth of the hydrogen lines following the order of temperature, those of α And. being peculiarly thin. α Cyg. is classed A2 because of the clear beginnings of solar metallic lines and especially the calcium line (K). Its hydrogen lines are excessively fine, and this no doubt is to be associated with its change of place with respect to Vega in the temperature sequences.

But I shall not pursue the examination of the list into further detail. Indeed the paper is rather an examination of the field than a presentation of final determinations of stellar temperatures. For that reason I have not put the word in its title. At the present stage the measures of the photograms leave a good deal that I hope experience will amend.

It is a weak point of the discussion that the value of the development constant γ, upon which the whole scale depends, was adopted at a round number without independent verification. Some definite means should be found for assigning absolute values without adopting a temperature for Capella. The range of densities is too restricted. And there are numerous other points that have been mentioned as calling for further examination. It may perhaps be felt that publication might have been withheld until the problems which the work has suggested were advanced more definitely towards solution. But even in this exploratory stage the outcome seems to me to possess interest. It has hardly been shown before that we are in sight of useful stellar measures of intensity by photography. Not very much has been published on the subject, though a good many workers must have been engaged upon it. It is partly in the hope that I may get the advantage of their experience that I put forward my own preliminary discussion.

Solar and Terrestrial Magnetic Phenomena, 1913–1921.
By the Rev. A. L. Cortie, S.J., D.Sc., F.Inst.P.

The present paper is mainly a statistical study of observations derived from the Stonyhurst, the Tortosa, and the Kodaikánal Observatories, which concern the solar and the magnetic phenomena of the present solar cycle, that is from the last solar minimum in the year 1913. The aim and object of the paper is, by the collation and comparison of observations, to throw some light upon the questions, as to which class of solar phenomena is the more closely connected with magnetic disturbance, and as to the mode of the propagation of the influence, generally supposed to be electronic from the sun, which is operative in such disturbances.

1. *Sun-spots and Magnetic Disturbances.*—In the following table the mean daily disc-area of the spots is expressed in units of 1/5000 of the visible disc. The mean daily range of the horizontal magnetic direction (D) is given in minutes of arc, and that of the horizontal force (H) is units γ, where $\gamma \equiv 10^{-5}$ C.G.S. unit. The quiet magnetic days are five in each month.* With regard to the characters great (*g*), and very great (*v.g.*) attached to magnetic disturbances, the note *g* is given to a disturbance in which the excess of the range over the mean daily range for the quiet days of the month is between 15 and 20 minutes of arc. Similarly, if the excess is greater than 20 minutes of arc, the disturbance is marked as very great.

The columns headed D and H give the mean daily ranges for the quiet days only of the month, these headed D_1 and H_1 give the same ranges for all days of a month. The excesses $D_1 - D$ and $H_1 - H$ set out in succeeding columns for the different years, are evidently a truer gauge of relative magnetic activity than either D and H or D_1 and H_1.

* For the years 1913–1917, the quiet magnetic days were taken as 10 in each month.

TABLE I.

Year.	Mean Daily Spot Disc-Area.	Magnetic Mean Daily Range. Quiet Days. D.	Quiet Days. H.	All Days. D_1.	All Days. H_1.	Excess Ranges. D_1–D.	Excess Ranges. H_1–H.	Disturbances Character. *g.*	Disturbances Character. *v.g.*
1913	0·04	7·2	28	9·7	39	2·5	11	1	0
1914	0·82	7·2	34	10·2	47	3·0	13	3	0
1915	4·51	7·3	35	11·7	58	4·4	23	6	2
1916	4·52	8·1	42	12·1	63	4·0	21	4	1
1917	12·10	8·7	40	11·8	58	3·1	18	7	4
1918	7·90	5·8	39	12·4	69	6·6	30	13	8
1919	8·40	6·6	31	12·7	67	6·1	36	21	18
1920	4·05	6·1	28	11·2	57	5·1	29	15	13
1921	3·14	6·9	38	11·4	54	4·5	16	6	7

The maximum of spot activity occurred in the year 1917. As is shown markedly in the columns of the above table giving the excess of D_1 and of H_1 over D and H respectively, as also from the numbers of great and of very great disturbances, the maximum of relative magnetic activity took place in the years 1918 and 1919, immediately following that of greatest spot activity. In these years there were many active and large spots. But the spots had also approached nearer to the sun's equator. Consequently, since the earth's heliographic latitude varies within the limits of only $\pm 7^\circ$, it was more favourably situated relatively to the foci of solar disturbance in 1918 and in 1919, than in the year of greatest solar activity 1917. It was still more favourably situated in the years 1920, 1921. But, as the second column of the table shows, solar activity had then considerably decreased. At the same time the most active outbursts of spots in these two years were accompanied by magnetic storms of exceptional violence.

Hence we may conclude, as has been already deduced in my former papers on the relations of solar and terrestrial magnetic disturbance;* first, that magnetic disturbance in any sun-spot cycle is in general greater after than at the maximum and, secondly, that this is due to the more favourable positions of the centres of solar activity relatively to the earth. Such a position of advantage is in general a necessary condition for the precipitation of a magnetic disturbance.

Referring to the table, it will be noticed, with regard to the magnetically quiet days, that the mean daily ranges are at a maximum at the period of solar maximum. If the mean daily ranges of all days be taken into account, as in columns five and six, the maxima occur after the solar maximum.

The following is a summary account of the course of solar activity during the years under discussion, and of the corresponding magnetic disturbances. The period of greatest solar activity extended from 1917 February to the end of 1919 August.

* *Monthly Notices*, 73, 436; 76, 15, 16, 634.

The disc-area of the spots in 1913 was the smallest observed since 1898, when the comparative tabulations were commenced at Stonyhurst. A marked increase of solar activity began in 1917 February, and reached its greatest intensity in August, in which month the mean daily disc-area was 25 units, or about double of that of any other month in the year. The most active period was August 6–16, when the mean daily disc-area was 40 units. The maximum area on any one particular day was 50 units on August 11. It was at this period that the first of the four exceptionally great magnetic storms of the cycle occurred, on 1917 August 9, 10; renewed on August 13, 14. This 1917 August group was of exceptional size, and second to none that had appeared for the previous 38 years. The February group, too, attained a very great area, 37 units on February 10. It was accompanied on February 15 by a disturbance, marked *g*, but in no way abnormal.

Magnetically, too, 1917 was a relatively quiet year, and out of accord with the solar activity as represented by sun-spots. But at the actual maximum of solar activity in August, the mean daily range in H for the month was greater than that in any other month since, and including the last solar maximum in 1905. Also the mean daily range in D was greater than the greatest for the preceding seven years. On the other hand, the greatest monthly range was less in February, with mean daily spot-area 14·6, than in January with mean daily spot-area 5·6. This was true both for D and for H, the monthly ranges being for January, 57′, and 239 γ, and for February, 36′, and 142 γ.

The year 1918 was magnetically more active than 1917, though the sun-spot area had decreased. There was, however, no storm of exceptional violence.

In the year 1919 there was a revival of sun-spot activity, and large groups of sun-spots were observed in February, May, June, and particularly in August. In this last month a large triple equatorial group crossed the sun's disc, August 13–25, being central on August 19. This compound group extended 20° in longitude and 17° in latitude. The greatest disc-area of the month, 24·0 units, occurred on August 21. The group lasted for four solar rotations, but after the first appearance of this great group the spot-area steadily and continuously declined until the end of the year.

Another exceptionally violent magnetic storm occurred in August 11–12, heralding the approach of the complex sun-spot group, which was then very near the sun's E limb, though not visible. Otherwise magnetically there was very little difference between the years 1918 and 1919.

In 1920 there was a considerable decline in sun-spot activity. But there was one disturbed area, mean latitude −6°, which extended from longitude 114° to 150°, and which was active from 1919 December 27 to 1920 May 16.

Each appearance of the group was accompanied by a magnetic disturbance, which culminated in the very violent storm of March 22.

There was a further decline in sun-spot activity in the year 1921. But a large sun-spot group, on the solar equator, crossed the sun's disc May 8–21. It extended over 12° in longitude, and 6° in latitude, and attained a maximum disc-area, 16·5 units, on May 14. The passage of

this spot was accompanied by a whole series of magnetic disturbances, forming a protracted storm, recalling the similar set of disturbances in 1882 November. A very violent storm accompanied the meridian passage of the group on May 13–14.

Consequently three of the four magnetic storms of exceptional violence, namely those of 1917, 1920, and 1921, were accompanied by large active sun-spot groups, the largest in their respective years. But with regard to that of 1919 August 11–12, the connection with the greatest sun-spot composite group of the year was remote, being merely one of position, and connection with disturbances in the same zone of latitude.

This disturbed area of considerable dimensions, containing the largest group of the year, a triple equatorial group extending over 20° in longitude, did not appear on the sun's E limb until August 14.* It was central on August 19. The total disc-area of the group on this date was 19·6 units. The only magnetic disturbance of any consequence which occurred during the passage of this very large group across the sun's disc was on August 19, character g, with ranges $D = 17'$ and $H = 147\ \gamma$.

The spot, which was apparently more directly connected with the magnetic storm, was a relatively small one in latitude $-9°$ and in longitude 212°. Its maximum disc-area was only 3 units, and on August 11 and 12, it was 14° and 28° respectively to the west of the central meridian. On the drawings it showed much agitation and change visually between the dates August 10 and 11. And, according to the observations of Mr. Buss, it displayed extraordinary activity in the spectroscope, much more intense than that of the great sun-spot groups that followed it. These spectroscopic phenomena consisted in brilliant reversals in H_α, with simultaneous remarkable D_3 absorption effects. Then at a later stage H_α absorption effects, with deflections and displacements first to the violet and then to the red. The spot was a striking example of localised intensity of activity.

The same region had been a focus of spot-activity in May, June, and July, and a series of magnetic disturbances at approximately 27 days interval from May 24 to October 6 was most probably connected with this region of the sun. For the five recurrences of the magnetic disturbance in a total of 135 days give a mean synodic rotation period for the focus of activity of 27·00 days, or a sidereal period of 25·14 days. This corresponds to a daily solar rotation period of 14°·32, which is that of latitude $\pm 5°{\cdot}7$. This is sufficiently concordant with the mean latitude of the successive sun-spot groups $-7°{\cdot}8$ to point to a connection with them. We may therefore conclude that it was the smaller active spot, and not the great group, which was more immediately connected with the violent magnetic storm of August 11–12.

Except for the occasional appearance of some small and very small groups (Nos. 1302, 1317, 1322, 1357, 1360 of the *Tortosa Observatory Bulletins*), this belt between solar latitudes $-5°$ and $-10°$ remained undisturbed until the appearance of the group of 1919 December 27, in latitude $-5°{\cdot}5$ and longitude 112°, which culminated in the very large

* *British Association Report, Edinburgh*, 1921, p. 416. The conclusion there stated needs modification by this more detailed discussion.

group of disc-area 34 units, which crossed the sun in 1920 March 16–29 (*M.N.R.A.S.*, **80**, 574). This great group was accompanied by the violent magnetic storm of 1920 March 22. It is noticeable that the same belt of latitude, in semi-equatorial regions, which had been disturbed concomitantly with a series of magnetic disturbances from 1919 May 24 to October 6 should, a second time, after a period of four quiet rotations, be associated with a still longer series lasting from 1920 January 1 to November 21.

Judging from the longer series of such magnetic disturbances, the solar activity would appear to come in a succession of pulses with intervals of quiescence. The following consideration lends countenance to this view. In my paper in *Monthly Notices*, **82**, 170, a table was given of the longer series of terrestrial magnetic disturbances associated with the active sun-spot groups during the present solar cycle.

Mr. C. P. Butler, of the Solar Physics Observatory, Cambridge, has pointed out that there is a "further striking relation in the sequence of dates, as all the intermediate intervals are also multiples of the solar synodic rotation, or so near as to be taken as such. So that it looks as if the whole period of disturbance was a function of the solar synodic rotation." We may rearrange the table as follows:—

TABLE II.

Principal Magnetic Series.

No.	Year.	Duration.	No. of Days.	Mean Period.	Intervals.	No. of Days.	Mean Period.	
1	1917	July 12–Nov. 26	137	27·40				
*2	1918	July 1–Jan. 8	191	27·29	Nov. 26–July 1	217	27·13	
3	1919	May 24–Oct. 6	135	27·00	Jan. 8–May 24	136	27·20	
4	1920	Jan. 1–Nov. 21	325	27·08	Oct. 6–Jan. 1	87	29·50	26·60
5	1921	March 22–Sept. 29	191	27·29	Nov. 21–March 22	121	24·20	
6	1921	May 21–Dec. 24	217	27·13	Nov. 21–May 21	181	25·86	

That is, the intervals between the disturbed periods are approximately multiples of the solar synodic rotation period. As far as sun-spots are concerned, attention was called to this alternation of quiet and disturbed periods in terms of synodic solar rotation periods, as long ago as the year 1900, in my paper on "The Duration of the Greater Sun-spot Disturbances for the Years 1881–99," *M.N.*, **60**, 531 *et seq.*

2. *Calcium Flocculi and Magnetic Disturbances.*—The statistics contained in the following table have been derived, for the sun-spots and for the calcium flocculi, from the monthly and annual bulletins of the observatory of the Ebro, Tortosa, and for the excess magnetic ranges from the Stonyhurst records. The areas of the sun-spots are expressed in units of a millionth, and of the flocculi in one hundred millionth of the sun's visible hemisphere. The excess magnetic ranges are in minutes of arc for D, and in units γ for H.

* In my former table there was an arithmetical error in series No 2. The interval is 191 and not 218 days as printed. The correction, which will affect also the numbers in the other columns, does not alter the mean sidereal period 25·31 days, as given at the foot of the table.

TABLE III.

Calcium Flocculi and Excess Magnetic Ranges.

Year.	Mean Daily Areas.		Mean Distance from Sun's Equator.		Mean Daily Excess Ranges.	
	Spots.	Flocculi.	Spots.	Flocculi.	D.	H.
	10^6.	10^6.	°	°	′	γ
1913	6	27	22·0	19·6	2·5	11
1914	126	198	22·2	23·0	3·0	13
1915	530	862	18·5	20·3	4·4	23
1916	526	802	15·8	17·3	4·0	21
1917	1272	981	14·7	15·6	3·1	18
1918	764	669	12·9	13·4	6·6	30
1919	844	670	10·8	12·3	6·1	36
1920	749	1634	11·0	12·7	5·1	29
1921	541	907	8·9	10·0	4·5	16

The question has been raised as to whether the calcium flocculi are connected with magnetic disturbance. As a secondary phenomenon, concomitant with sun-spots, they are; but as a primary condition for magnetic disturbance they are probably not. The table shows that, except for the year 1920 when there was an extraordinary development in mean daily area, the flocculi both in area and in position follow the sun-spots.

Compact bright calcium flocculi, like the faculæ which are the lower levels of the flocculous clouds, closely surround a sun-spot in the earlier period of its life-history. As the spot group develops, the flocculi extend about it in ever-increasing area, and when the spot has disappeared the faculæ and flocculi cover very great areas, but in a fainter and diffuse condition. Consequently we may first make a comparison between the flocculi without spots, though not necessarily independent of spots, and those which accompany spots. The ratios for the different years are given in the second column of the next table.

TABLE IV.

Frequency Ratios of Calcium Flocculi.

Year.	Without Spots. / With Spots.	Numbers.		d/c
		Condensed.	Diffuse.	
1913	4·58	89	65	0·8
1914	2·19	204	194	0·9
1915	1·08	312	432	1·4
1916	1·21	525	722	1·4
1917	0·62	790	643	0·8
1918	0·94	613	679	1·1
1919	0·79	431	507	1·2
1920	1·44	157	872	5·5
1921	1·98	143	676	4·7

From this it follows, that the calcium flocculi accompanying spots were relatively more frequent than those without spots, in the years of maximum sun-spots. But, as the spots and the flocculi moved towards the sun's equator, the flocculi without spots were gradually increasing in number though, except for 1920 and 1921, diminishing in area.

As the magnetic disturbances were also increasing in number, it might appear at first sight as though the two phenomena were primarily connected.

But besides numbers and areas it is necessary to consider also the character of the flocculi. At Tortosa they are divided into three classes or types—condensed, fairly condensed, and diffuse. For the present purpose, omitting the intermediate type, it will be sufficient to institute a comparison between the condensed and the diffuse types. The numbers are given for each year in the third and fourth columns of the table and a frequency ratio in the fifth column. From this it appears that in the years of minimum sun-spots the flocculi are few and mainly confined to those immediately surrounding the spots. That is, they die out before they spread very far. In the maximum year of spots, 1917, the condensed flocculi, which are generally associated with spots, are more frequent than the diffuse.

After the maximum diffuse flocculi preponderate in increasing numbers. Judging from the analogous spreading of faculæ, this is undoubtedly the effect of the numerous sun-spots that have died out. In 1920, the year of maximum area of flocculi, the diffuse flocculi are more than five times more frequent than the condensed flocculi. Such fainter diffuse objects, the embers of a series of outbursts, can presumably have but little effect on the frequency of magnetic storms. In this year the numbers and the excess ranges in magnetic disturbances were decreasing.

But in the years 1918, 1919, immediately after the sun-spot maximum, the case is different. In these years the magnetic disturbances were increasing in numbers, as were also the excess ranges in D and H. When a group of sun-spots has disappeared, the region which it had occupied remains magnetically active, often for several solar rotations. The flocculi mark such regions, and link up successive outbursts of sun-spots. Therefore, we may conclude, concomitantly and in connection with sun-spots, faculæ and calcium flocculi are associated with magnetic disturbances, and would as a related phenomenon be the condition for long series of magnetic disturbances, in terms of the synodic rotation period of the sun.

3. *Prominences and Magnetic Disturbances.*—In the following table statistics of solar prominences, both in mean daily profile area, in square minutes of arc, and of mean daily numbers, as also of H_α absorption markings in mean daily area, in terms of the one-millionth of the sun's visible hemisphere, are collected and compared. The figures have been derived from those published in the *Bulletins of the Kodaikánal Observatory*. Columns are added from the Stonyhurst records for mean daily sun-spot disc-areas, and for mean daily excess ranges in magnetic horizontal direction and force.

TABLE V.

Year.	Kodaikánal Mean Daily. Prominences. Profile-Area. Square \	Prominences. Numbers.	Hα Absorptions. Area 10^6.	Stonyhurst Mean Daily. Sun-Spots. Disc-Area 1/5000.	Excess. D. \	Ranges. H. γ.
1913	2·17	16·97	94	0·04	2·5	11
1914	3·14	20·13	...	0·82	3·0	13
1915	5·28	18·64	2542	4·51	4·4	23
1916	3·77	18·97	1710	4·52	4·0	21
1917	5·16	19·82	3246	12·10	3·1	18
1918	4·12	17·05	3610	7·90	6·6	30
1919	3·71	12·48	3496	8·40	6·1	36
1920	4·30	14·56	4817	4·05	5·1	29
1921	4·09	14·14	3584	3·34	4·5	16

The mean daily areas and numbers of the bright prominences follow closely those of the sun-spots for the successive years. But not so the areas of the absorption markings which are relatively more numerous in the years 1920, 1921, than in the years of maximum sun-spots 1917–1919.

In the memoir on "Results of Prominence Observations," by Mr. and Mrs. Evershed,* it is stated in the summary that there are four belts round the sun, two in the north and two in the south, which are specially marked by prominences. Of these, two are high-latitude belts and two are low or mid-latitude belts, which are distinguished by the production of prominences of different forms. With regard to the low-latitude belts the prominences in them act sympathetically with the sun-spots, in their rise to maximum and fall to minimum area, as also in their gradual approach to the sun's equator, corresponding to the diminution in the sun-spot activity. But, with the exception of the metallic prominences, and the small and low prominences of the "rocket" type, the greater proportion of these prominences have no direct connection with sun-spots. This is more particularly verified in the case of the larger prominences, of which there are many in these low-lying zones. These occur in regions free from sun-spots. When dense enough they are observed projected on the sun's disc as dark filaments. While "arches," "rockets" and "metallic jets" are the types of prominences which are associated with sun-spots, the characteristic types of prominence in the high-latitude zones are "pyramids," and "columns."

Again the movement of these high-latitude prominences is not towards the solar equator, but towards the poles, which are attained about the time of sun-spot maximum, when the prominences disappear. After sun-spot maximum the prominences form again in latitude $\pm 50°$, the greatest activity being reached between maximum and minimum. Seemingly there are two currents in evidence, the one flowing polewards and the other concomitantly with the sun-spots flowing towards the equator.

* *Memoirs of the Kodaikánal Observatory*, vol. i., part ii., pp. 123–124.

As corroborative evidence of these conclusions we may note the results set forth in a recent memoir* from the Kalocsa Observatory, embodying the observations of Father Fenyi, S.J., extending over a period of thirty-two years.

With regard to magnetic storms the authors of the Kodaikánal memoir state: "When prominences reach the poles, magnetic storms on earth reach their maximum frequency; but this is also the time of sun-spot maximum." We think that this last statement needs modification, as the magnetic disturbances are most frequent in the years following the maximum, when the mean latitudes of the spots are in semi-equatorial regions on the sun. They continue: "On the whole, the relation of magnetic storms seems to be closer with sun-spots than with prominences. It may be, however, that one of the necessary conditions for a sun-spot to have power to cause a magnetic storm on earth is the presence above it of an eruptive prominence."

Great magnetic storms are connected with disturbed and active sun-spots. A sun-spot group is most disturbed when it consists of two main spots with several spots between them, in the earlier phase of its life-history. At such times the spectroscope shows very great activity, with numerous reversals and distortions of the H_α line, in the region between the two main spost. Inasmuch as eruptive prominences are connected with such sun-spots, they are an indication of intense disturbance of the region. In this sense, concomitantly with the spot-outburst, they may be a necessary condition for a great terrestrial magnetic storm.

But whether prominences, independently of sun-spots can be effective in the production of magnetic storms is highly problematical. The evidence is rather to the contrary. In a former paper on "Solar Prominences and Terrestrial Magnetism,"† I have discussed this question, making a detailed study of prominences and magnetic disturbances for the years 1887–1888. The conclusion arrived at was that no certain relation could be established between prominences and magnetic storms, "seeing that in the cases of both eruptive and non-eruptive prominences, though some few seem to be connected with magnetic storms, others equally high and equally active were totally unconnected with any extraordinary movements of the needles."

Many photographs of the solar corona—we may instance in particular those taken by Campbell 1918 June 8—have shown the very close connection which exists between the interlacing arches of the lower reaches of the corona, the long streamers, and the prominences.‡ But the varying types of the corona of maximum, minimum, and intermediate years demonstrate the concomitant movement of the streamers in solar latitude with the sun-spots.

Very rarely, if ever, is the outbreak of a sun-spot disturbance an isolated phenomenon. The various sun-spot groups are connected one with the other, and an area of disturbance is generally so affected for a

* "Die Periodicität der Protuberanzen" nach Beobachtungen, in Kalocsa in den Jahren 1886–1917, von J. Fenyi, S.J., Kalocsa, 1922.

† *Astrophysical Journal*, **18**, 287–293, 1903 Nov.

‡ "On the Relationship between Solar Prominences and the Corona," W. J. S. Lockyer, *M.N.*, **82**, 323.

considerable period. These various spots are seemingly the photospheric manifestations of deep-seated foci of disturbance. The general connection between all classes of solar phenomena, spots, faculæ, flocculi, prominences, and coronal streamers is undoubted. But there is a particular and more intimate connection between spots, faculæ, calcium flocculi, and the shorter streamers of the solar corona. It is the sun-spot which is the centre of disturbance, and it is with the sun-spot that the magnetic disturbances are more directly associated. When a magnetic disturbance occurs without the presence of a sun-spot, it will be generally found to be a member of a series, which was originally and primarily connected with a sun-spot, the solar region of disturbance being marked by faculæ and calcium flocculi. When there is a sun-spot without a magnetic storm, it will be found either that it is a spot which has passed its maximum phase of activity, a large round regular spot for instance, or a spot which occurs in latitudes too high or too low for the electronic emanations to affect the earth. Position relatively to the earth, as well as size and activity, is a determining factor in the production of a magnetic storm.

4. *The Dark H_α Markings.*—We have seen, with regard to bright prominences, that the greater proportion, even of those occurring in the lower lying zones, have no direct connection with sun-spots. The denser prominences when observed projected on the sun's surface appear as dark H_α absorptions. These denser prominences also are those of the mid-latitude and equatorial zones. In the period 1917–1919, for instance,* the polar prominences and high-latitude prominences were much less marked by corresponding H_α markings than those in mid-latitudes. The inference is that the polar prominences, though so conspicuous in number and profile-area at solar maximum, must be of low density. The markings showed a tendency to move towards the solar equator in 1919, and their mean latitude in 1920 was about $\pm 20°$. At that period, too, they attained their maximum mean daily area, as is shown in column four of Table V.

As with the calcium flocculi so with the dark absorption markings, there is a marked increase in area after the sun-spot maximum has passed. This increase corresponds, too, with a greater magnetic activity. Comparing the year of maximum sun-spot activity of the present cycle, 1917, with the year 1920, while there was a diminution of 66 per cent. in the disc-area of spots (Stonyhurst), or 40 per cent. if the areas be corrected for foreshortening (Tortosa), there was an increase of 67 per cent. in the area of the calcium flocculi, also corrected for foreshortening. Similarly, although the mean daily profile-area of the bright prominences diminished 17 per cent., the mean daily area of the dark H_α markings increased 48 per cent. From which it would appear, that a diminution in the area of the bright prominences was accompanied by a great increase in their density, so as to render them visible as H_α markings on the sun's disc.

With regard to the mean positions of the H_α dark markings, there are two zones, N. and S., similar to those of the bright prominences. The two higher belts, between $\pm 50°$ and $\pm 70°$, which were in evidence

* *Kodaikánal Bulletins.*

in 1915, disappeared after 1917 June, except for a small peak in the curve of areas in the S. hemisphere. The lower zones were occupied by H_α markings, which generally had a maximum about 10° higher than the mean latitudes of the sun-spots. As the solar cycle advanced they moved towards the solar equator in sympathy with the sun-spots. In the period 1915, January–June, the H_α markings in the higher latitudes formed a belt "approximately parallel to the equator, and where the speed of the solar rotation is slower. Both these facts make the prominence due to an H_α marking in these latitudes endure for a larger number of days" (*Kodaikánal Bulletin*, **47**, 139). Again in the period 1916 July–December, there was an increase of activity in the H_α markings in the N. hemisphere, which was contrary to every other form of prominence activity. In the period 1917, July–December (*Bulletin*, 58, 245), there is noted a large increase in areas and numbers of H_α markings, although the prominences at the limb showed a decrease of mean area. "This indicates an increase of density of the prominences, excepting only those in the high-latitude zones of activity above latitude 60°, which have seldom given evidence of their presence on the disc" (*loc. cit.*).

While, therefore, with the advance of the solar cycle, the sun-spots diminish progressively in area, and in their mean distance from the sun's equator, they are accompanied in this progressive movement towards the equator by the calcium flocculi, which increase greatly in area. They are followed also, but in a higher mean latitude, by the H_α absorption markings, indicative of the denser prominences as seen projected on the sun's disc, which increase both in number and in mean daily area, as the spot-area diminishes. These phenomena, occurring after the sun-spot and bright prominence maximum, when magnetic disturbances, too, become more frequent, must be noted. Nevertheless, the connection between areas on the sun disturbed by sun-spots, and in many cases between individual sun-spots and magnetic disturbances, in by far the greater number of cases, is too close to warrant any other conclusion than that the sun-spots mark the chief centres of solar activity influencing magnetic disturbance. The other phenomena, bright prominences, calcium flocculi, and H_α absorption markings, although they may influence the solar action as conditioning magnetic disturbance, especially after the sun-spot maximum, are but concomitant and secondary.

Summary.

1. The magnetic disturbances were greatest in the years immediately following sun-spot maximum, when the mean latitudes of the spots were in semi-equatorial regions.

2. The magnetic storms of very great violence in 1917, 1920, 1921, were accompanied by the largest sun-spot groups in each year.

3. The great storm of 1919 was immediately connected with a small spot, and only remotely with the great group of the year.

4. An active sun-spot belt between 5° and 10° S. latitude was associated with long 27-day series of magnetic disturbances in 1919, 1920.

5. The solar activity coincident with such series comes in a succession of pulses, separated by intervals which are a function of the sun's synodical rotation.

6. Calcium flocculi are connected with series of magnetic disturbances as a secondary condition. They link together successive outbursts of spots, and maintain the magnetic activity of a region when the spots have disappeared.

7. The mean latitudes of limb and disc prominences are much higher than those of the spots, and except as a concomitant condition, they are not directly connected with magnetic disturbances.

8. But the great increase in both calcium flocculi and H_α markings in the later years of the cycle, when magnetic disturbances were also on the increase, is to be noted.

9. The longer coronal streamers are connected with prominences, but there are streamers marking disturbed sun-spot areas on the sun, which have been associated with series of magnetic disturbances.

Stonyhurst College Observatory:
1922 *July* 30.

Comparison of Sun-spot Areas and Terrestrial Magnetic Horizontal Force Ranges, 1911–21. By Rev. B. G. Swindells, S.J., B.Sc., A.R.C.Sc. (Plate 11.)

(*Communicated by Father Cortie.*)

The accompanying curves are based entirely on the results of observations published in the annual reports of the Stonyhurst Observatory for the years 1911–21 inclusive. They must be regarded as presenting only a general, although a true, view of the changes in the respective phenomena which have occurred during those years. In the first place, in regard to the mean daily sun-spot areas for each month, in the winter especially, these are at times based on comparatively few measurements. In 1914 January, for instance, the number of days on which the sun was observed only amounted to seven, and the same was true of 1913 January. Fortunately, at this period of minimum sun-spot activity, the area of the sun-spots observed was constantly extremely small, and consequently for the purposes of the curves no great error can have been introduced. In the years 1917, 1918, the lowest numbers of observations in any one month were eight in 1917 November, and nine in 1918 February. But at the period of maximum, 1917 June–August, the greatest periods of no observations were two days (August 8, 9), and three days (August 28–30).

A more serious uncertainty results from the measurements themselves. On instituting a comparison between these measurements and the published Greenwich measurements for corresponding days, a considerable divergence was occasionally noted, which could not be accounted for by the difference between the photographic and the visual-drawn images. In general, it is found that the Stonyhurst measurements are less than those of Greenwich. This is a characteristic of both Brother McKeon's

measurements, ending with 1919, and the measurements of the present observer, and is in all probability due to the difference in the images as photographed and as drawn. But sometimes Brother M°Keon's measurements differed from those at Greenwich by an amount in excess of this characteristic difference, which does not exceed as a rule $\frac{1}{5000}$th of the sun's disc, and is normally considerably less. As Brother M°Keon was a very accurate observer, it is possible that in such cases the spots were drawn and photographed during their period of rapid development. Much, too, depends on the type of the spot-groups measured, whether diffuse or condensed.* It did not seem worth the trouble to try to correct these Stonyhurst readings, which are apparently in error, first because a general conspectus was all that was required, and, secondly, because such accidental differences would be largely obliterated in the process of smoothing, which was applied to the results.

This process of smoothing affords a third reason for regarding the curves as a general and not as a detailed summary of the phenomena, while at the same time it ensures the probable truth of this summary by eliminating accidental errors. The smoothing consists in the double application of the process by which for each number in the series is substituted the mean of that number and of the two adjacent to it. If there was only one adjacent number, as at the ends of the series, the mean of the two was taken.

The magnetic curves exhibit the variations in the mean daily range of the horizontal magnetic intensity for each month, first when only the quiet days are taken into account, and, secondly, when all days are included. From 1911 to 1918 the ten quietest days of each month were utilised; from 1919 onwards, in accordance with international usage, the five quietest days. The curves have been subjected to the same process of smoothing as the sun-spot area curve. The difference between the ordinates of these two magnetic curves obviously affords an indication of the amount of magnetic disturbance produced by any outside influences. The difference is exhibited in the third magnetic curve of excess range, viz. all days minus quiet days. In regard to this curve much caution must be exercised. It is to be noted that it represents the difference of a difference, for the range is itself a difference of measured magnitudes. Consequently, in the mere process of formation a possible error is introduced, fourfold that of the measured magnitudes themselves. If we assume these correct to the nearest unit, then the possible error of the excess range is two units. This renders any small variations in the curve, even after smoothing, of no value as a basis of deductions, as is discovered on calculating the curve by different methods. And the magnitudes of the ordinates of the curve are never correct to one unit, and probably are not correct to two units. Such as the curve is, however, it gives a fair representation of the amount of extra disturbance of the needle, and the approximate time of its occurrence.

Having described the limitations of the curves, we may attempt to draw some deductions from their general character. Taking them in order from the lowest upwards, we are at once struck with the regularity of the annual period in the variation of the force on quiet days. We

* Atmospheric conditions would also make a difference.—A. L. C.

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also notice that the eleven-year period is well brought out, the yearly minima and the maxima of the curves rising and falling in a regular manner, the period of which coincides with that of the eleven years of the records under consideration. The difference between successive yearly maxima and minima remains approximately constant, with the exception of the years 1912 and 1917, when the variation in range becomes comparatively small.

In the curve of the range for all days, we observe that from 1912 to 1917 it follows fairly closely the form of the lower curve. It reaches its maximum in 1917 August, the month of maximum solar activity, in a sharp peak. In 1915 there is a small irregularity, but in 1918 the shape of the curve becomes markedly irregular, this irregularity lasting until 1920 inclusive.

These features are again brought out in the excess curve, which has notable maxima in 1915 and 1917, and rises to a considerable height in 1919.

An examination of the sun-spot curve reveals some very remarkable coincidences. The increase in sun-spot area in the later months of 1912, which, though slight absolutely, is great in relation to the quiet state of the sun, coincides with a disturbance in the magnetic field revealed by the excess curve. Another disturbance is noticeable in 1914, coincident with a solar outbreak. But that of 1915 is rendered remarkable by the existence of an imposed decrease of disturbance, more or less coincident with the rise to maximum of the solar activity. The solar maximum of 1917 produces an effect at the time of its occurrence, as is shown in the two middle curves; but the chief magnetic disturbance occurs two years later, when the sun, though still active, is considerably past its maximum activity, as measured in actual magnitude. But, though this is true, it must also be noticed that the sun was persistently active until the last few months of 1919.

In attempting to compare more accurately the times of occurrence of the various maxima, the limitations of the curves must be borne in mind. The times of maxima of the excess curve may be out by as much as two or three months, specially in the cases where the curvature is relatively small. It is possible that the small maximum in the excess curve in 1914 does not correspond to any real phenomenon. Taking these limitations into account, however, it seems that the magnetic disturbances, in the two years before the solar maximum of 1917, preceded the corresponding solar disturbances; the magnetic and solar disturbances coincide at the time of the 1917 maximum, but in the two subsequent years the magnetic disturbances follow on the corresponding solar disturbances. The divergence in time decreases until approximate coincidence is again attained in 1921.

No explanation of this relation, nor of the curious decrease of magnetic activity in 1915, which exceeds in magnitude any possible errors of the curve, is at present offered. My thanks are due to Father Cortie for suggesting, and for helping in the present investigation.

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1922 *August* 30.